MAY 2022 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. State Cauchy's theorem for higher derivatives.
2. Define pole.
3. Define simply connected region.
4. Define residue.
5. Find the pole of $f(z) = \frac{1}{\sin z}$.
6. Write the Poisson's formula in cartesian coordinates.
7. Write Taylor's series for $f(z)$.
8. State Weierstrass theorem.
9. Prove that $\Gamma(z+1) = z\Gamma(z)$.
10. Define gamma function.
11. Define isolated singularity.
12. Write mean value property.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. State and prove Liouville's theorem.

14. State and prove Rouche's theorem.

15. State and prove the maximum principle.

16. Write down the Taylor's series of the following functions at the origin at the origin.

    (a) e^z

    (b) $\sin z$

    (c) $\log(1+z)$.

17. Prove that the necessary and sufficient condition for the absolute convergence of the product $\prod_1^{\infty}(1+\alpha_n)$ is the convergence of the series $\sum_1^{n}|a_n|$.

18. State and prove the argument principle.

19. Find the poles and residues of $\frac{1}{z^2+5z+6}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Suppose that $\phi(\zeta)$ is continuous on the arc γ. Then prove that the function $F_n(z) = \int_\gamma \frac{\phi(\zeta) d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative $F_n'(z) = nF_{n+1}(z)$.

21. State and prove the residue theorem.

22. Derive the Poisson's formula in polar co-ordinates.

23. Prove that the arithmetic mean of a harmonic function over the concentric circles $|z| = r$ is a linear function of $\log r$,

$$\frac{1}{2\pi}\int_{|z|=r} u d\theta = \alpha \log r + \beta$$

and if u is harmonic in a disc $\alpha = 0$ and the arithmetic mean is a constant.

24. State and Jensen's formula.

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