MAY 2023 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define index of a point with respect to γ.

2. Define removable singularity.

3. Define simply connected region.

4. Define residue.

5. Define harmonic function.

6. Find the poles of $\frac{1}{z^2+5z+6}$.

7. State reflection principle.

8. Write the Poisson integral of U.

9. Define gamma function.

10. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$.

11. Define meromorphic function.

12. If γ is the circle $|z|=1$, then find the value of $n(\gamma,2)$.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. State and prove the Cauchy's integral formula.

14. State and prove the argument principle.

15. If u_1 and u_2 are harmonic in a region Ω, then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$.

16. State and prove Hurwitz theorem.

17. Prove that $\Gamma(z)\Gamma(1-z) = \dfrac{\pi}{\sin \pi z}$.

18. State and prove Cauchy's theorem for higher derivatives.

19. State and prove Cauchy's inequality.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove Taylor's theorem.

21. If $pdx + qdy$ is locally exact in Ω, then prove that $\int_\gamma pdx + qdy = 0$ for every cycle $\gamma \sim 0$ in Ω.

22. Derive the Poisson's formula in cartesian coordinates.

23. Derive Laurent's series.

24. State and prove Mittag-Leffler's theorem.

———————