MAY 2024 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define index of a point with respect to γ.

2. Define removable singularity.

3. Define locally exact differential.

4. Define cycle.

5. Define harmonic function.

6. Find the pole of $f(z)=\dfrac{1}{\sin z}$

7. State reflection principle.

8. Write the Poisson integral of *U*.

9. Prove that $\Gamma(z+1)=z\Gamma(z)$.

10. Write the value of $\Gamma(5)$.

11. Define meromorphic function.

12. Is the series $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots$ converges? Justify your answer.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. State and prove maximum principle.

14. State and prove Rouche's theorem.

15. Prove that the arithmetic mean of a harmonic function over the concentric circles $|z| = r$ is a linear function of log r, $\frac{1}{2\pi}\int_{|z|=r} u d\theta = \alpha \log r + \beta$ and if u is harmonic in a disc $\alpha = 0$ and the arithmetic mean is a constant.

16. State and prove Hurwitz theorem.

17. Prove that $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$.

18. State and prove Cauchy's theorem for higher derivatives.

19. State and prove Liouville's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

21. State and prove the residue theorem.

22. Derive the Poisson's formula in polar co-ordinates.

23. State and prove Schwarz theorem.

24. State and prove Jensen's formula.

———————