MAY 2025 **P/ID 37511/PMA3L**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Prove that $n(-\gamma, a) = -n(\gamma, a)$.

2. Define meromorphic function.

3. When a cycle is said to be homologous to zero?

4. Define Chain.

5. Define harmonic function.

6. Find the poles of $\frac{1}{(z^2-1)^2}$.

7. Write the Poisson integral of U.

8. State Hurwitz theorem.

9. Define infinite products

10. Define gamma function,

11. Define residue.

12. Write conjugate differential of du.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. State and prove Cauchy's integral formula.

14. State and prove Weierstrass theorem on essential sigularities.

15. State and prove argument principle.

16. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

17. Find the poles and residues of $\frac{1}{z^2 + 5z + 6}$.

18. Prove that Laurent's development is unique.

19. Prove that $\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$ and deduce $\Gamma\left(\frac{1}{2}\right)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Suppose that $\phi(\zeta)$ is continuous on the arc γ. Then prove that the function

$F_n(z) = \int_\gamma \frac{\phi(\zeta)d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative $F_n'(z) = nF_{n+1}(z)$

21. State and prove Cauchy's theorem.

22. Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$ where $a > 1$

23. State and prove Weierstrass theorem.

24. Prove that $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$.