DECEMBER 2022 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define topology.

2. Define closure.

3. Define connected space.

4. Define path connected.

5. When a subset A of a space X is said to be dense in X?

6. Define normal space.

7. Define box topology.

8. Define projection mapping.

9. Define homotopic functions.

10. Define fundamental group.

11. Define continuous function.

12. Define homeomorphism.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If $\mathfrak{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $\mathfrak{D} = \{B \times C / B \in \mathfrak{B} \text{ and } C \in \mathcal{C}\}$ is a for the topology of $X \times Y$.

14. Prove that the image of a connected space under a continuous map is connected.

15. Suppose that X has a countable basis, then prove that every open covering of X contains a countable subcollection covering X.

16. Let X be a set, let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.

17. Prove that the map $\hat{\alpha}$ is a group isomorphism.

18. Let X be a ser. Let $\mathfrak{B}$ be a basis for a topology τ on X. Then prove that τ equals the collections of all unions of elements of $\mathfrak{B}$.

19. State and prove the pasting lemma.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let A be as subset of the topological space X.

(a) The prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A.

(b) Supposing the topology of X is given by a basis, then prove that $x \in \overline{A}$ if and only if every basis element B containing x intersects A.

21. State and prove the intermediate value theorem.

22. Prove that the subspace of a Hausdorff space is Hausdorff and the product of Hausdoff spaces is Hausdorff.

23. State and prove Tychonoff theorem.

24. Prove that the relations $\simeq$ and $\simeq_p$ are equivalence relations.

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