DECEMBER 2023 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a topology on a Set X.

2. Using topologies on X, Y construct a topology on the Cartesian product $X \times Y$.

3. Define a Hausdorff space.

4. Show that the only connected subsets of Q are the singletons.

5. State the intermediate value theorem.

6. State the Lebesgue number lemma.

7. Give one example for each:
   (a) First countable space.
   (b) Second countable space.

8. Define the box topology on the Cartesian product $X_1 \times X_2 \ldots \times X_n \ldots = \prod_{i \in N} X_i$.

9. Define a path homology between two paths in X.

10. Verify the equality $[f]*[g]=[f\times g]$.

11. State the group axioms satisfied by the elements of $\pi_1(X, x)$ under *.

12. Define a contractible space. Give an example.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let A' – set of all limit points of A in X. Explain $\overline{A}$ using A'.

14. Let $f : A \to X \times Y$ be defined as $f(a)=(f_1(a), f_2(a))$ where $f_1 : A \to X$ $f_2 : A \to Y$.

Prove that f is continuous if and only if f_1, f_2 are continuous.

15. Prove that the finite product of connected spaces is connected.

16. Explain path components, connected components of a topological space X. When they are same?

17. Show that a subspace of a regular space is regular, and also show that a product of regular spaces is regular.

18. Let X be a set. Let D be a collection of subsets of X that is maximal with respect to finite intersection property. Prove that

(a) Any finite intersection of elements of D is an element of D.

(b) If $A \subset X$ and A intersects every element of D, then A is an element of D.

19. Verify the associativity property on π, (X, x) state all the necessary facts to prove this.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. (a) Prove that a finite point set in a Hausdorff space in closed.

(b) Let X – space satisfying T_1 axiom. Let $A \subset x$. Then $x \in A$ is a limit point of A if and only if every neighbourhood of x contains infinitely many prints of A – verify.

(c) X – Hausdorff, prove that any sequence in X converge to atmost one pant of X.

21. Let X be a simply ordered set having the least upper bound property. Show that for the order topology on X, each closed interval in X is compact.

22. State and prove Uryshon's Metrization theorem.

23. (a) Explain the box topology on $\{X_\alpha\}_{\alpha \in J}$.

(b) Using the collection $S = \bigcup_{\beta \in J} S_\beta$ construct a topology on $\prod_{\alpha \in J} X_\alpha$.

(c) Compare these two topologies.

(d) Prove that $\prod x_\alpha$ is Hausdroff if each X_α is.

24. (a) Prove $\simeq p$ is an equivalence relation.

(b) In $R^2 \setminus \{0\}$ give two paths which are not straight line path homotopic. Explain.

(c) Prove $\hat{\alpha}$ is an isomorphism.

(d) h_* is an isomorphism if $h : X \to Y$ is a homeomorphism. Verify.

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