DECEMBER 2024 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Give examples of two different topologies on a set with four elements.

2. Define a open map.

3. Show that a subspace of a Hausdorff space is Hausdorff.

4. Give an example of a basis for $\mathbb{R}$ under standard topology.

5. Show that $[0,1] \cup [3,4]$ is compact.

6. State the Lebesque Number Lemma.

7. Define a completely regular space.

8. Define a normal space.

9. Verify that $[0,1]$ is path connected.

10. Define a connected component of a topological space. Is it closed? Give reasons.

11. How many non-isomorphic fundamental groups are there for S^1 based at different points.

12. Define a star convex space. Give example.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Construct a basis for the product space $X \times Y$ using the basis of X and Y and a sub basis using the topologies on X and Y.

14. For a subset A of X. Prove that $\overline{A} = A \cup A'$.

15. State and prove the intermediate value theorem.

16. Show that a space is locally path connected if and only if for each open $U \subset X$, each component of U is open in X.

17. Prove that, if a space has a countable basis, then every open covering of X, has a countable subcollection covering X.

18. Explain the Box topology and the product topology on the product space.

19. Prove that the path homotopy is an equivalence relation.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Explain the topologies $\mathbb{R}_l$, $\mathbb{R}_k$ and compare them with standard topology on $\mathbb{R}$.

21. Prove that the product of finitely many compact spaces is compact.

22. State and prove Urysohn lemma.

23. State and prove Tychonoff theorem.

24. Prove that the product of paths operation $*$ satisfies associativity property.

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