MAY 2022 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a basis for a topology.

2. Define a homemorphism between topological spaces.

3. Characterize the connected subsets of $\mathbb{R}$.

4. Define a path connected space.

5. State the second countability axiom.

6. Define a regular space.

7. Define a box topology.

8. Define a product topology. Give an example.

9. Define the fundamental group of a topological space X at a point $x_0 \in X$.

10. Define a simply connected space. Is $\mathbb{R}$, with usual topology a simply connected space?

11. Define a limit point of a set in a topological space.

12. Define Hausdorff space, verify $\mathbb{R}$, is a Hausdorff space with usual topology.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let $\mathfrak{B}$ and $\mathfrak{B}'$ be basis for the topologies τ and τ', respectively, on X. Prove that the following are equivalent.

(a) τ' is finer than τ.

(b) For each $x \in X$ and each basis element $B \in \mathfrak{B}$ containing x, there is a basis element $B' \in \mathfrak{B}'$ such that $x \in B' \subset B$.

14. Prove that every closed subspace of a compact space is compact.

15. Prove that every metrizable space is normal.

16. Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod_{\alpha \in J} X_\alpha$ have the product topology. Then prove that the function f is continuous if and only if each function f_α is continuous.

17. Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic. Explain why the circle S' is not simply connected.

18. Prove that every finite point set in a Hausdorff space X is closed. Give an example of a topological space which is not Hausdorff.

19. State and prove the pasting lemma.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then prove that $\mathcal{C}$ is a basis for the topology of X.

21. If L is a linear continuum in the order topology, then prove that L is connected and so are intervals and rays in L.

22. Prove that a subspace of a regular space is regular and the product of regular spaces is regular.

23. State and prove Tychonoff theorem.

24. Prove that the relations $\simeq$ and $\simeq_p$ are equivalence relations.

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