MAY 2024 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Construct a topology on the set $\{1, 2, 3, 4\}$.

2. Define a continuous function. What is topological property?

3. What is a limit point? Prove that A in X is closed if and only if it contains all its limit points.

4. Prove that $X \times Y$ is connected if X, Y are connected.

5. Define a path connected space.

6. Define a compact subset of a topological space.

7. State the first countability and the second countability axioms.

8. State the Tychonoff theorem.

9. When do we say two continuous maps $f : X \to Y$, $g : X \to Y$ are homotopic?

10. State the group axioms satisfied by the elements of $\pi,(X,x)$ under $*$.

11. Let α be a path in X from x to y. Define $\alpha : \pi_1(X,x) \to \pi_1(X,y)$. Check if it is well defined.

12. Verify that the unit ball B^2 in $\mathbb{R}^2$ has trivial fundamental group.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Define a sub basis for a topological space X. Construct the sub basis for the product topological space $X \times Y$.

14. Let $A \subset X$. Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A.

15. State and prove the intermediate value theorem.

16. Every compact subspace of a Hausdroff space is closed.

17. Show that a subspace of a completely regular space is completely regular. Prove that product of completely regular spaces is completely regular.

18. Define a box topology for the family $\{X_\alpha\}_{\alpha\in I}$. Using the projections, construct a product topology for the family $\{X_\alpha\}_{\alpha\in I}$ Compare these two topologies if I is a finite set.

19. Define h_* where $h : (X_1X_0) \to (Y_1Y_0)$ continuous. If h is a homomorphism, verify h_* is an isomorphism.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. (a) Explain with example finite complement topology.

(b) Explain the topological spaces $\mathbb{R}_l$ $\mathbb{R}_k$.

(c) Relate $\mathbb{R}_l$ $\mathbb{R}_k$ with standard topology on $\mathbb{R}$.

21. (a) Check if $\mathbb{R}$ is compact

(b) $X = \{0\} \cup \{ 1/n | n \in \mathbb{Z}_+ \} \subset \mathbb{R}$. Is X compact?

(c) Prove that the product of two compact spaces is compact.

22. State and prove Urysohn lemma.

23. Prove the Tychonoff theorem.

24. Verify all the group axioms for the elements of $\pi_1(X, x)$.
