MAY 2025 **P/ID 37512/PMA3M**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Show that $(-1, 1)$ and $\mathbb{R}$ are homeomorphic?

2. Find the closure of $\mathbb{Q}$.

3. Define a basis for a topology.

4. Define a Hausdorff space.

5. Show that $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}$.

6. Show that the set $\{1, 2, 3, 4\}$ is compact.

7. Define box topology.

8. When do you say, two subsets A and B of X are separated?

9. Verify that $[0, 1]$ is path connected.

10. Define a product topology for the family of topological spaces.

11. In $\mathbb{R}^2 \setminus \{0\}$. Give two paths which are not path homotopic.

12. Define a contractible space.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let $f : X \to Y$ be a function. Show that f is continuous if and only if for all closed subsets C in Y, $f^{-1}(C)$ is closed in X.

14. For a subspace Y of X. Prove that a set A in Y is closed if and only if $Y = C \cap Y$, where C is closed in X.

15. Define a connected space. Prove that the image of a connected space is connected.

16. State and prove the intermediate value theorem.

17. Prove that, if a space has a countable basis, then every open covering of X, has a countable subcollection covering X.

18. Explain the finite intersection property, countable intersection property.

19. Prove that existence of inverse for the product operation $*$ on the paths on X.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove all the rules for constructing new continuous functions.

21. Prove that the product of finitely many compact spaces is compact.

22. State and prove Urysohn Metrization theorem.

23. State and prove Tychonoff theorem.

24. Explain h_* where $h : X \to Y$ is continuous. Prove that it is an isomorphism if X is homoeomorphism to Y.

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