DECEMBER 2022 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. State the theorem on absolute probability.

2. Find the probability that heads appear twice when a coin is tossed three times.

3. Define quantile of order p.

4. Define a degenerate distribution.

5. State Cramér-Wold theorem.

6. State the properties of characteristic function.

7. Define Pólya distribution.

8. Define a two-point distribution.

9. Define gamma distribution.

10. Define uniform distribution.

11. State de Moivre-Laplace theorem.

12. Define stochastically convergent to zero.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. If the events A and B are independent, then prove that the events $\overline{A}$ and $\overline{B}$ are also independent.

14. Prove that the variance of the sum of an arbitrary finite number of independent random variables, whose variances exist, equals the sum of their variances.

15. Find the characteristic function of the random variable X having density

$$f(x) = \begin{cases} 0 \ \ for\ x < 0 \\ 1 \ \ for\ 0 \le x \le 1 \\ 0 \ for\ x > 1 \end{cases}$$

16. The joint distribution of the random variable (X, Y) is given by the density

$$f(x, y) = \begin{cases} \frac{1}{4}\left[1 + xy(x^2 - y^2)\right] for\ |x| \le 1 |y| \le 1, \\ 0 \qquad\qquad\qquad \text{for all other points} \end{cases}$$

Check whether the random variables X and Y are independent.

17. The random variable X has the distribution $N(1; 2)$. Find the probability that X is greater than 3 in absolute value.

18. State and prove the Bernoulli law of large numbers.

19. State and prove Borel-Cantelli Lemma

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Derive the distribution function of the product $Z = X\,Y$ of two random variables X and Y.

21. Find the two regression curves for the two dimensional normal distribution.

22. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a+h$ and $a-h (h>0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h) - F(a-h) = \lim_{T\to\infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sinh t}{t}\, e^{-ita} \phi(t)\, dt\,.$$

23. Discuss the Bernoulli scheme and derive the Binomial distribution.

24. State and prove the Lindberg-Lévy theorem.
