DECEMBER 2023 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions

1. Find the probability that heads atleast appear twice when a coin is tossed three times.

2. Define a degenerate distribution.

3. Define quantile of order p.

4. State Cramer-Wold theorem.

5. Write the Lapunov inequality.

6. Define a two-point distribution.

7. Define gamma distribution.

8. State and prove the Khintchin's law of large numbers.

9. Define Cauchy distribution.

10. State Bernoulli's law of large numbers

11. State de Moivre-Laplace theorem.

12. State Borel-Cantelli Lemma.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Let $\{A_n\}$, $n = 1, 2, \cdots$, be nondecreasing sequence of events and let A be their alternative. Prove that.

$$P(A) = \lim_{n \to \infty} P(A_n).$$

14. Prove that the variance of the sum of an arbitrary finite number of independent random variables, whose variances exist, equals the sum of their variances.

15. Find the characteristic function and the moments of a normal distribution.

16. Compute the semi-invariants of the Poisson distribution.

17. The random variable X has the distribution. $N(1;2)$. Find the probability that X is greater than 3 in absolute value.

18. An unbiased coin is tossed 100 times. Find the probability that heads will appear more than 50 times and less than 60 times.

19. State and prove the Kolmogorov inequality.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Derive the distribution function of the product $Z = X\,Y$ of two random variables X and Y.

21. The random variables X and Y have the joint density given by

$$f(x,y) = \frac{1}{2\pi}\ \exp\left(-\frac{x^2 - 2xy + 2y^2}{2}\right)$$

Find the coefficient of correlation.

22. If the l-th moment m_l of a random variable exists, prove that $m_l = \dfrac{\phi^{(l)}(0)}{i^l}$,s

where $\phi^{(l)}(0)$ is l-th derivative of the characteristic function $\phi(t)$ of this random variable at $t = 0$.

23. Discuss the Bernoulli scheme and derive the Binomial distribution.

24. State and prove Lévy–Cramér theorem.

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