MAY 2022 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Compute the probability that heads appear atleast twice in three successive tosses of a coin.

2. State the theorem on absolute probability.

3. Write the Lapunov inequality.

4. Define quantile of order p.

5. The random variable X can take on the values $x_1 = -1$, $x_2 = +1$ with probabilities $P(X - 1) = P(X = +1) = 0.5$. Find the characteristic function of X.

6. State Cramér-Wold theorem.

7. Define a two-point distribution.

8. Define probability generating function.

9. Define uniform distribution.

10. Define Cauchy distribution.

11. Define stochastically convergent to zero.

12. State Borel-Cantelli Lemma.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let $\{A_n\}$, $n = 1, 2, \cdots$, be non decreasing sequence of events and let A be their alternative. Prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

14. Suppose that the random variable X may take on two values $x_1 = 5$ and $x_2 = 10$ with probabilities $P(X = 5) = \frac{1}{3}$ and $P(X = 10) = \frac{2}{3}$. Find the distribution of the random variable $Y = 2X$.

15. Prove that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, equals the product of the expected values of these variables.

16. Compute the semi-invariants of the Poisson distribution.

17. Given $f(y) = 6y(1 - y)$ for $0 < y < 1$. What is the probability that Y is not greater than 0.2?

18. Let $F_n(x)$ $(n = 1, 2, \cdots)$ be the distribution function of the random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & for \quad x \le 0 \\ 1 & for \quad x > 0 \end{cases}.$$

19. State and prove the Kolmogorov inequality.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Derive the distribution function of the sum $Z = X + Y$ of two random variables X and Y.

21. Prove that $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

22. If the l-th moment m_l of a random variable exists, prove that $m_l = \dfrac{\phi^{(l)}(0)}{i^l}$, where $\phi^{(l)}(0)$ is the l-th derivative of the characteristic function $\phi(t)$ if this random variable at $t = 0$.

23. Derive the characteristic function of the Cauchy distribution.

24. State and prove de Moivre-Laplace theorem.

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