MAY 2023 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Find the probability that heads appear twice when a coin is tossed three times.

2. State the theorem on absolute probability.

3. Define a degenerate distribution.

4. Define quantile of order p.

5. The random variable X can take on the values $x_1 = -1,\ x_2 = +1$ with probabilities $P(X = -1) = P(X = +1) = 0.5$. Find the characteristic function of X.

6. State the properties of characteristic function.

7. Define probability generating function.

8. Define Pólya distribution.

9. Define a two-point distribution.

10. Define gamma distribution.

11. Define stochastically convergent to zero.

12. State de Moivre-Laplace theorem.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Let $\{A_n\}, n = 1,2,.....,$ be nonincreasing sequence of events and let A be their product. Prove that $P(A) = \lim_{n\to\infty} P(A_n)$.

14. Suppose that the random variable X may take on two values $x_1 = 5$ and $x_2 = 10$ with probabilities $P(X = 5) = \frac{1}{3}$ and $P(X = 10) = \frac{2}{3}$. Find the distribution of the random variable $Y = 2X$.

15. Find the characteristics function of the random variable X having density

$$f(x) = \begin{cases} 0 \ \ for \quad x < 0 \\ 1 \ \ for \quad 0 \le x \le 1 \\ 0 \ \ for \quad x > 1 \end{cases}$$

16. Find the density function of the random variable X whose characteristics function is

$$\phi(t) = \begin{cases} 1 - |t| & for \ |t| \leq 1 \\ 0 & for \ |t| > 1 \end{cases}$$

17. Given $f(y) = 6y\,(1-y)$ for $0 < y < 1$. What is the probability that Y is not greater than 0.2?

18. State and prove the Bernoulli law of large numbers.

19. State and prove the Khintchin's law of large numbers.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Derive the distribution function of the sum $Z = X + Y$ of two random variables X and Y.

21. Find the two regression curves for the two dimensional normal distribution.

22. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristics function of the random variable *X*. If $a+h$ and $a-h\ (h>0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h)-F(a-h)=\lim_{T\to\infty}\frac{1}{\pi}\int_{-T}^{T}\frac{\sin ht}{t}e^{-ita}\ \phi(t)\,dt.$$

23. Derive the characteristic function of the Cauchy distribution.

24. State and prove the Lindberg-Lévy theorem.

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