MAY 2024 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. State the theorem on absolute probability.
2. Find the probability that heads appear twice when a coin is tossed three times.
3. The random variable X can take on the values $x_1 = -1$, $x_2 = +1$ with probabilities $P(X = -1) = P(X = +1) = 0.5$. Find the characteristic function of X.
4. Define a degenerate distribution.
5. State the properties of characteristic function.
6. Define a two-point distribution.
7. Define Pólya distribution.
8. Define Cauchy distribution.
9. Write Kolmogorov inequality.
10. Define rectangular distribution.
11. State Khintchin's law of large numbers.
12. Define stochastically convergent to zero.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If the events A and B are independent, then prove that the events $\overline{A}$ and $\overline{B}$ are also independent.

14. State and prove Lapunov inequality.

15. Prove that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, the product of the expected values of these variables.

16. The joint distribution of the random variable (X, Y) is given by the density

$$f(x, y) = \begin{cases} \frac{1}{4}\left[1 + xy\left(x^2 - y^2\right)\right] & \text{for} \quad |x| \le 1 \ |y| \le 1, \\ 0 & \text{for} \quad \text{all other points} \end{cases}$$

Check whether the random variables X and Y are independent.

17. State and prove Poisson's theorem.

18. Let $F_n(x)(n=1, 2,....)$ be the distribution function of the random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & \text{for} \quad x \le 0 \\ 1 & \text{for} \quad x > 0 \end{cases}$$

19. State and prove Borel-Cantelli Lemma.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If X and Y are independent standard normal variates, find the distribution of $Z = X + Y$.

21. Prove that $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

22. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a+h$ and $a-h \quad (h>0)$ are continuity points of the distribution function $F(x)$, prove that

$$F(a+h) - F(a-h) = \lim_{T\to\infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sin ht}{t} e^{-ita} \phi(t) dt .$$

23. Find the first and second moments of the Pólya distribution.

24. State and prove de Moivre-Laplace theorem.