MAY 2025 **P/ID 37514/PMA3P**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. State the theorem on absolute probability.

2. Define an impossible event.

3. Define moment of order k.

4. Define standard deviation.

5. Define beta distribution.

6. Define two-point distribution.

7. Define probability generating function.

8. Define Cauchy distribution.

9. State Kolmogorov theorem.

10. State Chebyshev's theorem.

11. Define gamma distribution.

12. Define stochastically convergent to zero.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. If the events A and B are independent, then prove that the events $\overline{A}$ and $\overline{B}$ are also independent.

14. Suppose that the random variable X may take on two values $x_1 = 5$ and $x_2 = 10$ with probabilities $P(X=5)=\frac{1}{3}$ and $P(X=10)=\frac{2}{3}$. Find the distribution of the random variable $Y=2X$.

15. Prove that the variance of the sum of an arbitrary finite number of independent variables, whose variances exist, equals the sum of their variances.

16. Find the characteristic function of the random variable X having density

$$f(x)=\begin{cases}0 & \text{for} \quad x<0\\ 1 & \text{for} \quad 0\le x\le 1\\ 0 & \text{for} \quad x>1\end{cases}$$

17. The characteristic function of the random variable X is $\phi(t)=\exp\left(-t^2/2\right)$. Find the density of X.

18. An unbiased coin is tossed 100 times. Find the probability that heads will appear more than 50 times and less than 60 times.

19. State and prove Lindeberg-Lévy theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove Baye's theorem.

21. Prove that $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(Y = aX + b) = 1$ to hold.

22. If the l-th moment m_l of a random variable exists, prove that

$$m_l = \frac{\phi^{(l)}(0)}{i^l},$$

where $\phi^{(l)}(0)$ is the l-th derivative of the characteristic function $\phi(t)$ of this random variable at $t = 0$.

23. State and prove Poisson theorem.

24. State and prove De Moivre Laplace theorem.

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