DECEMBER 2022 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Riemann zeta function.

2. Define normal family.

3. Write Schwarz-Christoffel's formula.

4. Write the mean-value property.

5. Define elliptic function.

6. Define canonical basis.

7. Define Weierstrass elliptic function.

8. Define the sigma function $\sigma(z)$.

9. Define direct analytic continuation.

10. Define germs.

11. Define sheaf.

12. Define Weierstrass zeta function $\zeta(z)$.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

14. Derive Harnack's inequality.

15. Show that any two bases of the same module are connected by a unimodular transformation.

16. Show that a non-constant elliptic function has equally many poles as it has zeros.

17. Prove that $\zeta(z+w_1) = \zeta(z) + \eta_1$.

18. Prove that $\sigma(z+w_1) = -\sigma(z)\, e^{\eta_1\left(z+\frac{w_1}{2}\right)}$.

19. Prove that two analytic continuations $\overline{\gamma}_1$ and $\overline{\gamma}_2$ of a global analytic function f along the same arc γ are either identical, or $\overline{\gamma}_1(t) \neq \overline{\gamma}_2(t)$ for all t.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove the Arzela's theorem.

21. State and prove the Riemann mapping theorem.

22. Prove that the zeros $a_1, a_2, .., a_n$ and poles $b_1, b_2, .., b_n$ of an elliptic function satisfy $a_1 + a_2 + ... + a_n \equiv b_1 + b_2 + ... + b_n \pmod{M}$.

23. Show that any even elliptic function with periods w_1, w_2 can be expressed in the form $c\prod_{k=1}^{n} \frac{\wp(z) - \wp(a_k)}{\wp(z) - \wp(b_k)}$ (c- constant), provided that 0 is neither a zero nor a pole.

24. State and prove the *Monodromy* theorem.

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