DECEMBER 2023 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Riemann ζ-function.

2. Define equicontinuons function.

3. Define univalent function.

4. Write mean value property for harmonic functions.

5. Define elliptic function.

6. Define unimodular transformation

7. Define Weierstrass zeta function.

8. Define sigma function.

9. Define direct analytic continuations.

10. Define germs.

11. Write the complex Fourier development of $f(z)$.

12. Define canonical basis.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that $\int_0^\infty \frac{x^{s-1}}{e^x - 1} dx = \zeta(s)\,\Gamma(s)$.

14. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

15. Prove that a continuous function $u(z)$ which satisfies $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})\, d\theta; |z - z_0| \le r$ is necessarily harmonic.

16. Show that a non-constant elliptic function has equally many poles as it has zeros.

17. Prove that $\zeta(z + w_1) = \zeta(z) + \eta_1$.

18. Prove that $\eta_1 w_2 - \eta_2 w_1 = 2\pi i$.

19. Prove that the homotopy classes of closed curves from z_0 with respect to the region Ω forms a group.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that a family $\mathfrak{F}$ is totally bounded if and only if every compact subset $E \subset \Omega$ and every $\in > 0$, there exist $f_1, f_2, ..., f_n \in \mathfrak{F}$ such that if $f \in \mathfrak{F}$, then $d(f, f_j) < \in$ for some f_j.

21. State and prove the Riemann mapping theorem,

22. Prove that a discrete module consists either of zero alone, of the integral multiples nw of a single complex number $w \neq 0$, or of all the linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with non-real ratio $\frac{w_2}{w_1}$.

23. Obtain the differential equation satisfied by Weierstrass elliptic function $\wp(z)$.

24. State and prove the *Monodromy* theorem.

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