DECEMBER 2024 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Riemann ζ-function.

2. Define normal family.

3. Define univalent function.

4. What is free boundary arc?

5. Define periodic function.

6. Define canonical basis.

7. Define Weierstrass zeta function.

8. Define σ-function.

9. Define global analytic function.

10. Define homotopic curves.

11. Write the complex Fourier development of $f(z)$.

12. Define period module.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that $\int_0^\infty \frac{x^{s-1}}{e^x - 1} dx = \zeta(s)\Gamma(s)$.

14. State and prove Harnack's inequality.

15. Show that any two bases of the same module are connected by a unimodular transformation.

16. Prove that $\wp(z+u) = -\wp(z) - \wp(u) + \frac{1}{4}\left[\frac{\wp'(z) - \wp'(u)}{\wp(z) - \wp(u)}\right]^2$.

17. Prove that two analytic continuations $\bar{\gamma}_1$ and $\bar{\gamma}_2$ of a global analytic function f along the same arc γ are either identical, or $\bar{\gamma}_1(t) \neq \bar{\gamma}_2(t)$ for all t.

18. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

19. Prove that the zeros $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy $a_1 + a_2 + \ldots + a_n \equiv b_1 + b_2 + \ldots + b_n \pmod{M}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that a family $\mathfrak{F}$ is totally bounded if and only if every compact subset $E \subset \Omega$ and every $\in > 0$, there exist $f_1, f_2, \ldots, f_n \in \mathfrak{F}$ such that if $f \in \mathfrak{F}$, then $d(f, f_j) < \in$ for some f_j.

21. Derive Schwarz-Christoffel's formula.

22. Show that any two bases of the same module are connected by a unimodular transformation.

23. Obtain the differential equation satisfied by Weierstrass elliptic function $\wp(z)$.

24. State and prove the *Monodromy* theorem.

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