MAY 2022 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define equicontinuous function.

2. Define normal family.

3. Write the mean-value property.

4. State Harnack's inequality.

5. Define period module.

6. Define elliptic function.

7. Define Weierstrass zeta function $\zeta(z)$.

8. Define the sigma function $\sigma(z)$.

9. Define global analytic function.

10. Define germs.

11. Define sheaf.

12. Define canonical basis.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that a family $\Im$ is totally bounded if and only if every compact subset $E \subset \Omega$ and every $\in > 0$, there exist $f_1, f_2, \cdots, f_n \in \Im$ such that if $f \in \Im$, then $d(f, f_j) <\in$ for some f_i.

14. Prove that a continuous function $u(z)$ which satisfies the mean value property is necessarily harmonic.

15. Show that a non-constant elliptic function has equally many poles as it has zeros.

16. Prove that an elliptic function without poles is a constant.

17. Prove that $\eta_1 w_2 - \eta_2 w_1 = 2\pi i$.

18. Prove that $\sigma(z + w_1) = -\sigma(z) e^{\eta_1\left(z+\frac{w_1}{2}\right)}$.

19. Prove that two analytic continuations $\overline{\gamma}_1$ and $\overline{\gamma}_2$ of a global analytic function **f** along the same are γ are either identical or, $\overline{\gamma}_1(t) \neq \overline{\gamma}_2(t)$ for all *t*.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If $\{p_n\}$ is an ascending sequence of prime numbers and if $\sigma = \mathrm{Re}(s) > 1$, prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty} \left(1 - p_n^{-s}\right)$.

21. Derive Schwarz-Christoffel's formula.

22. Prove that a discrete module consists either of zero alone, of the integral multiples nw of a single complex number $w \neq 0$, or of all the linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with non-real ratio $\frac{w_2}{w_1}$.

23. Obtain the differential equation satisfied by Weierstrass Elliptic function $\wp(z)$.

24. State and prove the *Monodromy* theorem.

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