MAY 2023 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define equicontinuous function.
2. Define normal family.
3. Define univalent function.
4. Write mean value property for harmonic functions.
5. Define periodic function.
6. Define unimodular transformation.
7. Define σ–function.
8. Define Weierstrass zeta function $\zeta(z)$.
9. Define global analytic function.
10. Define sheaf.
11. Write the complex Fourier development of $f(z)$.
12. Define canonical basis.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If $\{p_n\}$ is an ascending sequence of prime numbers and if $\sigma = \mathrm{Re}(s) > 1$, prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}\left(1 - p_n^{-s}\right)$

14. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

15. State and prove Harnack's inequality.

16. Prove that the sum of the residues of an elliptic function is zero.

17. Discuss the pseudo-periodicity of $\zeta(z)$.

18. Obtain Legendre's relation.

19. Prove that two analytic continuations $\overline{\gamma}_1$ and $\overline{\gamma}_2$ of a global analytic function f along the same arc γ are either identical, or $\overline{\gamma}_1(t) \neq \overline{\gamma}_2(t)$ for all t.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that a family $\mathcal{F}$ is normal if and only if its closure with respect to the distance function $\rho(f, g) = \sum_{k=1}^{\infty} \delta_k(f, g) 2^{-k}$ is compact.

21. Derive Schwarz-Christoffel's formula.

22. Prove that there exists a basis (w_1, w_2) such that the ratio $\tau = \frac{w_2}{w_1}$ satisfies the following conditions.

(a) Im $\tau \geq 0$

(b) $-\frac{1}{2} < \text{Re } \tau \leq \frac{1}{2}$

(c) $|\tau| \geq 1$

(d) Re $\tau \geq 0$ if $|\tau| = 1$

Also prove the uniqueness of ratio τ, which is determined by the above four conditions.

23. Derive the equation $\wp(z) = \frac{1}{z^2} + \sum_{w \neq 0} \left(\frac{1}{(z-w)^2} - \frac{1}{w^2} \right)$

24. State and prove the *Monodromy* theorem.

———————