MAY 2024 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define equicontinuous function.

2. Define normal family.

3. Define univalent function.

4. Write mean value property for harmonic functions.

5. Define period module.

6. Define canonical basis.

7. Prove that $\varsigma'(z) = -\wp(z)$.

8. Define Weierstrass elliptic function.

9. Define function element.

10. Define homotopic curves.

11. Define germs.

12. Write the complex Fourier development of $f(z)$.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that $\int_0^\infty \frac{x^{s-1}}{e^x - 1} dx = \zeta(s)\Gamma(s)$.

14. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

15. Prove that a continuous function $u(z)$ which satisfies

$$u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u\left(z_0 + re^{i\theta}\right) d\theta; |z = z_0| \leq r$$

is necessarily harmonic.

16. Show that any two bases of the same module are connected by a unimodular transformation.

17. Prove that the zeros $a_1, a_2, ..., a_n$ and poles $b_1, b_2, ..., b_n$ of an elliptic function satisfy $a_1 + a_2 + \cdots + a_n \equiv b_1 + b_2 + \cdots + b_n (\text{mod } M)$.

18. Prove that $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$.

19. Prove that two analytic continuations $\bar{\gamma}_1$ and $\bar{\gamma}_2$ of a global analytic function f along the same arc γ are either identical, or $\bar{\gamma}_t(t) \neq \bar{\gamma}_2(t)$ for all t.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove Arzela- Ascoli's theorem.

21. Stat and prove the Harnack's principle.

22. Prove that a discrete module consists either of zero alone of the integral multiples $\eta\omega$ of a single complex number $\omega \neq 0$, or of all the linear combinations $n_1\omega_1 + n_2\omega_2$ with integral coefficient of two numbers ω_1, ω_2 with non- real ratio $\frac{\omega_2}{\omega_1}$.

23. Prove that $\wp(z) - \wp(u) = \frac{\sigma(z-u)\sigma(z+u)}{[\sigma(z)]^2[\sigma(u)]^2}$.

24. State and prove the *Monodromy* theorem.

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