MAY 2025 **P/ID 37516/PMA4S**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define equicontinous function.

2. Define normal family.

3. Define univalent function

4. Write mean value property for harmonic functions.

5. Define period module.

6. Define elliptic function.

7. Define σ function.

8. Prove that $\zeta'(z) = -\wp(z)$.

9. Define direct analytic continuations.

10. Define germs.

11. Write the complex Fourier development of $f(z)$.

12. Define periodic function.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. If $\{p_n\}$ is an ascending sequence of prime numbers and if $\sigma = \mathrm{Re}(s) > 1$ prove that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}\left(1 - p_n^{-s}\right)$.

14. Prove that a continuous function $u(z)$ which satisfies $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u\left(z_0 + re^{i\theta}\right)d\theta$; $|z - z_0| \le r$ is necessarily harmonic.

15. Show that a non-constant elliptic function has equally many poles as it has zeros.

16. Prove that $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$.

17. Prove that the homotopy classes of closed curves from z_0 with respect to the region Ω forms a group.

18. Prove that a locally bounded family of analytic functions has locally bounded derivatives.

19. Prove that the sum of the residues of an elliptic function is zero.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that the family $\mathcal{F}$ is normal if and only if its closure with respect to the distance function $\rho(f,g)=\sum_{k=1}^{\infty}\delta_k(f,g)2^{-k}$ is compact.

21. State and prove the Harnack's principle.

22. Prove that there exists a basis (ω_1,ω_2) such that the ratio $\tau=\frac{\omega_2}{\omega_1}$ satisfies the following conditions.

(a) $\operatorname{Im}\tau>0$

(b) $-\frac{1}{2}<\operatorname{Re}\tau\le\frac{1}{2}$

(c) $|\tau|\ge 1$

(d) $\operatorname{Re}\tau\ge 0$ if $|\tau|=1$.

Also prove the uniqueness of ratio τ, which is determined by the above four conditions.

23. Obtain the differential equation satisfied by Weierstrass elliptic function $\wp(z)$.

24. State and prove the *Monodromy* theorem.

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