DECEMBER 2023 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define a unit speed curve. Write down the speed parametrisation of the circle with center (0, 0, 0). Verify it.

2. What is a reparametrisation? When does a regular curve has a unit-speed reparametrisation? Verify.

3. Find the curvature of the curve $\gamma(t) = \left(t, t^2, t+5\right)$.

4. Write down the surface parch for the unit sphere.

5. Define a tangent space for a surface.

6. Find the first fundamental form the right helicoid.

7. Compare the Christofell symbols for the saddle surface.

8. Compute the principal curvatures of the plane.

9. Prove any normal section of a surface is a geodesic.

10. Find all geodesics of the sphere.

11. For a surfaces revolution, write down the Gaussian Curvature.

12. Define the first fundamental form of a surface.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that a parametrised curve has a unit speed reparamentrisation if and if it is regular.

14. For a unit speed plane curve, derive the tangent vector equation in terms of turning angle.

15. Define $D_p f$ where $f : S_1 \to S_2$ be a smooth map. show that it is linear.

16. (a) Check if $x^2 + y^2 + z^4 = 1$ is compact.

(b) Show that any compact surface is orientable.

17. Let $f : S_1 \to S_2$ be a smooth map. Show that f is a local isometry if and only if the symmetric bilinear forms $<,>p,\ f^{*}<,>p$ on $T_p\ S_1$, are equal $\forall\ P \in S_1$.

18. Prove that there is a unique geodesic through any point of a surface in any given tangent direction.

19. Find a surface patch whose first and second fundamental forms are $du^2 + dv^2, -du^2$ respectively.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. For every smooth function $k : (a,b) \to R$, prove that there is a unit speed curve in plane whose signed curvature is k. Prove also that it is unique upto an isometry.

21. (a) Prove that the transition maps of a smooth surface are smooth.

(b) Let U_1, U_2 be open in $\mathbb{R}^2$. Let $\sigma : U_1 \to \mathbb{R}^3$ be a regular surface parch. Let $\Phi : U_1 \to U_2$ be a differentiation. Show that $\tilde{\sigma} = \sigma \circ \Phi$ is a regular surface parch.

22. (a) Find the second fundamental form of a surface of revolution.

(b) Compare $<<u,w>>, \quad u,w \in T_p\ S$.

23. (a) Derive Geodesic equations.

(b) Determine the geodesic on the unit sphere.

24. Prove that the Gaussian curvature of a surface to preserved by local isometrics.

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