DECEMBER 2024 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Smooth Curve. Give Example.

2. Define a curvature. Write down the curvature of the circular helix.

3. Define a smooth surface.

4. Define a tangent space of the smooth surface, Give an example.

5. Define the First Fundamental Form (FFF) of the surface. Write down the FFF of S^2.

6. Write down the Second Fundamental form of the surface of revolution.

7. Define a Geodesic of a surface.

8. What are the geodesics of the unit sphere?

9. Is there any surface with the FFF and SFF given as $du^2 + dv^2$ and $-du^2$ respectively?

10. Write down the formula for Gaussion Curvature if $E = 1$ and $F = 0$.

11. Define a reparametrization of a curve.

12. Define a principal normal of a smooth space curve.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Find the arc length of a logarithmic spiral.

14. Compute the curvature of the circular helix.

15. Prove that the unit speed space curve with constant curvature and zero torsion is part of a circle.

16. Prove that the map $D_P(f)$ is linear.

17. State and Prove the Euler theorem.

18. Derive the Geodesic equations.

19. Determine the connected compact surfaces with constant Gaussion curvature.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let $f : (a,b) \to \mathbb{R}$ be any smooth function. Prove that there exists a unique unit speed plane curve upto isometry, whose signed curvature is f.

21. Prove that a smooth map $f : S \to S'$ between surfaces is a local isometry if and only if $, < , > p = f^* < , > p$ for all $P \in S$.

22. For a surface patch $\sigma : U \to \mathbb{R}^3$ and closed disc R_δ (defined approximately), K, a Guassian curvature of σ Show that $| K | = \lim_{\delta \to 0} \frac{A_N(R_\delta)}{A_\sigma(R_\delta)}$.

23. Prove that a unit speed curve is a geodesic if and only if $\frac{d}{d\tau} \mathcal{L}(\tau) = 0$ when $\tau = 0$ for all families of curves γ^τ with $\gamma^0 = \gamma$.

24. Derive Codazzi Mainardi Equations and the Gauss equations.

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