MAY 2022 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Parametrised Curve.

2. Define Arc-length.

3. What is a surface?

4. Define area of part of surface patch.

5. Write the formula for the normal curvature.

6. Explain the Gaussian and mean curvatures.

7. Define Geodesic.

8. Write the geodesic coordinates.

9. Write the Gauss Equations.

10. Write the Codazzi-Mainardi Equations.

11. Write the Christoffel symbols.

12. Compute the curvature of a right circular helicoid.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Let $\gamma(t)$ be a regular curve in R^3. Then prove that its curvature is $k = \dfrac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}\|^3}$, where the $\times$ indicates the vector product and the dot denotes d/dt.

14. Let U and $\overline{U}$ be open subsets of R^2 and let $\sigma : U \to R^3$ be a regular surface patch. Let $\Phi : \overline{U} \to U$ be bijection smooth map with smooth inverse map $\Phi^{-1} : U \to \overline{U}$. Then prove that $\overline{\sigma} = \sigma \circ \Phi : \overline{U} \to U$ is regular surface patch.

15. Prove that the area of a surface patch is unchanged by reparametrisation.

16. Meusnier's Theorem : State and prove.

17. Prove that if S is a compact surface, there is a point P of S at which is Gaussian curvature K is > 0.

18. A curve γ on a surface S is a geodesic if and only if, for any part $\gamma(t) = \sigma(u(t), v(t))$ of γ contained in a surface path σ of S, then prove the following :

$$\frac{d}{dt}(E\dot{u} + F\dot{v}) = \frac{1}{2}(E_u\dot{u}^2 + 2F_u\dot{u}\dot{v} + G_u\dot{v}^2),$$

$$\frac{d}{dt}(F\dot{u} + E\dot{v}) = \frac{1}{2}(E_v\dot{u}^2 + 2F_v\dot{u}\dot{v} + G_v\dot{v}^2),$$

where $Edu^2 + 2Fdudv + Gdv^2$ is the fundamental form of σ.

19. Prove that any map of any region of the earth's surface must distort distances.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let γ be a regular curve in R^3 with nowhere vanishing curvature (so that the torsion τ of γ is defined). Then prove that the image of γ is contained in plane if and only if τ is zero at every point of the curve.

21. Explain the ruled surfaces in detail with examples.

22. Define Weingarten map. Find the matrix of $\omega_{P,S}$. Compute the Gaussian curvatures for surface of revolution.

23. Prove that there is an open subset U of R^3 containing (0, 0) such that $\sigma : U \to R^3$ is an allowable surface patch of S. Moreover, the first fundamental form of σ is $du^2 + G(u,v)dv^2$ where G is a smooth function on U such that $G(0,v) = 1$, $G_u(0, v) = 0$, when – ever $(0, v) \in U$.

24. State and prove Gauss's Remarkable theorem.

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