MAY 2023 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a regular curve. Write a formula of its arc length.

2. Find the curvature of a circular helix.

3. Derive the signed currvature.

4. What is a surface patch? When it is regular?

5. Find the tangent space for the right helicoid.

6. Compute the first fundamental form of unit sphere.

7. Write down the formulas for the Christoffel symbols.

8. Compare the principal curvatures of the saddle surface.

9. Write down the geodesic equations.

10. When is a unit speed curve on a surface is geodesic?

11. What is a geodesic local dygeomorphism?

12. Define the first fundamental from of a surface.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that a parametrised curve has a unit – speed reparametrisation if and only if it is regular.

14. For a unit speed reparamensation of the circle describe the signed curvature. What happens if it is negative?

15. Describe the tangent plane of a surface patch.

16. Show that a sphere is compact. Show that every compact surface is orientable.

17. Define a local isometry. Let $f: S_1 \to S_2$ smooth. Define $f^* <,\ >_p$. Show that it is a symmetric bilinear form on $T_p S_1$.

18. Any local isometry between surfaces takes geodesics to the geodesics.

19. Derive $K = \frac{-1}{2\sqrt{EG}} \left\{ \frac{\delta}{\delta a}\left(\frac{Gu}{\sqrt{EG}} \right) + \frac{\delta}{\delta v}\left(\frac{Ev}{\sqrt{EG}} \right) \right\}$ for $F = 0$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Derive the formulas of curvature and torsion for space curves.

21. (a) Show that a smooth map $f : S_1 \to S_2$ is a local isometry if and only if the forms $<,>_p$, $f^{*}<,>_p$ on $T_p S_1$ are equal for all $p \in S_1$.

    (b) Find the first fundamental from of a generalized cone.

22. (a) Derive the matrix of w_p, interms of $f_I^{-1} f_{II}$.

    (b) Evaluate H, K interms of $\{E, F, G, L, M, N\}$.

23. Show that a unit speed curve γ is a geodesic on a surface if and only if $\frac{d}{f\tau}(L(\tau)) = 0$ when $\tau = 0$, for all families of curves γ^{τ} with $\gamma^{o} = \gamma$.

24. Prove that the Gaussian curvature of a surface is preserved by local isometrics.

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