MAY 2024 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Check if the logarithmic spiral is a regular curve. Find its arc-length whenever it is defined.

2. What is a unit-speed reparametrisation of the curve? Write down the formula of the curvature of regular curve.

3. Find the curvature of the curve $\gamma(t) = (t, t^2, t+5)$.

4. What is a surface patch? When it is regular?

5. Define a tangent space for a subspace.

6. Compute the first fundamental form of cylinder.

7. Write down the formulas for Christofell symbols.

8. Derive the formulas for Gaussian and mean curvatures using principal curvatures.

9. Prove that geodesic has a constant speed.

10. For the plane find all the geodesics.

11. For a helicoid write down the Gaussian curvature.

12. Define the first fundamental form of a surface.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Derive the formula of the curvature for regular curves.

14. For a unit speed plane curve, derive the equation of tangent vector in terms of turning angle.

15. Let $f : S_1 \to S_2$ be a smooth map. Define $D_p f$. Show that it is linear.

16. Show that any sphere is compact. Show also that a compact surface is orientable.

17. Show that a smooth map $f : S_1 \to S_2$ is a local isometry if and only if the forms $< , >_p$ and $f^* < , >_p$ on $T_p S_1^{'}$ are equal $\forall P \in S_1$.

18. Any local isometry between surfaces takes geodesics to the geodesics.

19. Find a surface patch whose first and second fundamental forms are $du^2 + du^2, -du^2$ respectively.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. (a) For a regular curve in $\mathbb{R}^3$. With nowhere vanishing curvature, prove that it is contained in a plane if and only if the torsion is zero at every point of the curve.

(b) Derive the torsion formula for space curves.

21. (a) Explain the surface patches for a sphere.

(b) Explain the reasons for circular cone is not a surface.

22. (a) Derive the matrix of W_p interms of

$\mathcal{H}_{II}^{-1}$ $\mathcal{H}_{II}$

(b) Evaluate H, K using $\{E, F, G, L, M, N\}$.

23. (a) Derive Geodesic equations.

(b) Determine geodesic on the unit sphere.

24. Derive the gauss, codazzi-mainardi equations.

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