MAY 2025 **P/ID 37517/PMA4T**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a unit speed curve. Give a unit speed reparametrization of the circle of radius R centered at origin.

2. Define a turning angle of a unit speed curve.

3. Write down a parametrization of the torus.

4. Define a compact surface.

5. Define a local isometry between the surfaces.

6. Write down the Second Fundamental Form of the elliptic paraboloid.

7. State the geodesic equations.

8. When does a curve in a smooth surface, a geodesic?

9. Is there any surface with the FFF and SFF given as $du^2 + \cos^2 udv^2$ and $\cos^2 udu^2 + dv^2$ respectively?

10. Write down the formula for Gaussion Curvature for the surface of revolution.

11. Define a regular curve.

12. Write down the formula of the torsion of a circular helix.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Prove that any reparametrization of a regular curve is regular.

14. Derive the formula of the torsion for the unit speed curve.

15. Derive Frenet-Serret Equations.

16. Explain time surface patches of the Mobius strip. Why is it a non-orientable surface?

17. Compute the matrix of the Weingarten map $W_{P,S}$.

18. Prove that, the local isometry maps geodesics to geodesics.

19. Determine all the connected compact surfaces with constant Gaussian curvature.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Derive the equation $\dot{b} = -\tau n$ and compute the torsion of the curve $\gamma(t) = \left(\frac{4}{5}\cos t, 1 - \sin t, \frac{-3}{5}\cos t\right)$.

21. Compute the first fundamental form of the cylinder.

22. Prove that any connected surface in which every point is umbilic, must be an open subset of a plant or a sphere.

23. Prove that, a unit speed curve is a geodesic if and only if $\frac{d}{d\tau}\, \mathcal{L}\, (\tau) = 0$ when $\tau = 0$ for all families of curves γ^{τ} with $\gamma^{0} = \gamma$.

24. State and prove the Gauss theorem Egregium and derive the formula of Gaussian curvature in various cases.

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