MAY 2023 **P/ID 37519/PMA4V**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define holonomic constraint.

2. Define virtual displacement.

3. Write Lagrange's equations.

4. What are ignorable coordinates?

5. Write Euler-Lagrange equation.

6. State modified Hamilton's principle.

7. Define Hamilton's Principal function.

8. State Stakel's theorem.

9. Discuss homogeneous canonical transformation.

10. What is momentum transformation?

11. Define conservative force.

12. Write the Lagrange's bracket expression $[u, v]$ for two variables (u, v).

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Consider a particle which is constrained to move on a fixed circular path of radius a. The equation of constraint is $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} = a$. Transform from cartesian to generalized coordinates for the above.

14. Discuss briefly about Natural systems.

15. Find the stationary values of the function $f = z$, subject to the constraints $\phi_1 = x^2 + y^2 + z^2 - 4 = 0$ and $\phi_2 = xy - 1 = 0$.

16. Discuss about conservative systems and ignorable coordinates.

17. Prove that the transformation $Q = \frac{1}{2}\left(q^2 + p^2\right)$, $P = -\tan^{-1}\frac{q}{p}$ is canonical.

18. Obtain the equation of motion for the Kepler problem in the form $r^2\dot{\theta} = \beta$.

19. Explain about Pfaffian differential forms.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. A particle of mass m is suspended by a massless wire of length $r = a + b\cos wt \quad (a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

21. Define Routhian function and prove that

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_i}\right) - \frac{\partial R}{\partial q_i} = 0 \quad (i = k+1, \ldots, n).$$

22. Derive Hamilton's canonical equation of motion.

23. State and prove Jacobi's theorem.

24. Consider the transformation $Q = \log \frac{\sin p}{q}$, $P = q \cot p$. Obtain the four major types of generating functions associated with this transformation.

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