MAY 2024 **P/ID 37519/PMA4V**

Time : Three hours Maximum : 80 marks

PART A — $(10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define holonomic constraint.

2. Define virtual work.

3. What are ignorable coordinates?

4. Define Routhian function.

5. State modified Hamilton's principle.

6. Write the necessary and sufficient condition that a function f have a stationary value at q_0.

7. Define Hamilton's Principal function.

8. Write the Pfaffian differential form.

9. Write down the Poisson bracket expression.

10. What is momentum transformation.

11. State the principle of conservation of energy.

12. Write the Lagrange's bracket expression $[u,v]$ for two variables (u,v).

PART B — $(5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. Consider a particle which is constrained to move on a fixed circular path of radius a. The equation of constraint is $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} = a$. Transform from Cartesian to generalized coordinates for the above.

14. A particle of mass m can slide without friction on the inside of a small tube which is bent in the form of a circle of radius r. The tube rotates about a vertical diameter with a constant angular velocity w. Write the differential equation for this motion.

15. Find the stationary values of the function $f = z$, subject to the constraints $\phi_1 = x^2 + y^2 + z^2 - 4 = 0$ and $\phi_2 = xy - 1 = 0$.

16. (a) State Stackel's theorem.

(b) Write down the two Stackel conditions.

17. Prove that the transformation

$$Q = \frac{1}{2}\left(q^2 + p^2\right), \quad P = -\tan^{-1}\frac{q}{p}.$$

is Canonical.

18. Consider a particle which is constrained to move on a fixed circular path of radius a. The equation of constraint is $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} = a$. Transform from Cartesian to generalized coordinates for the above.

19. Use Poisson bracket to show the transformation $Q = \sqrt{e^{-2q} - p^2}$, $P = \cos^{-1}\left(pe^q\right)$ is canonical.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

20. A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t \quad (a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

21. Derive Lagrange's equations.

22. Derive Hamilton's canonical equation of motion.

23. Analyze the Kepler's problem by using Hamilton-Jacobi method.

24. Consider the transformation $Q = \log \frac{\sin p}{q}$, $P = q \cot p$. Obtain the four major types of generating functions associated with this transformation.

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