DECEMBER 2022 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Define statistic.

2. Define simple random sample.

3. Define non-central X^2.

4. Define parametric hypothesis.

5. Define likelihood function of the purely continuous Markov process.

6. Define consistent estimate.

7. Define an asymptotically unbiased estimate.

8. State Backwell Theorem.

9. Define simple hypothesis.

10. Define a biased test.

11. Define the *OC* function of the Sequential Probability Ratio Test.

12. Write the fundamental identity.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Prove that the sequence $\{F_n(t)\}$ of distribution functions of Student's t with n degrees of freedom satisfies for every t, the relation

$$\lim_{n\to\infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt\,.$$

14. Derive χ^2 independence test by contingency tables.

15. The random variable X has the Poisson distribution.

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda},\ (k = 0, 1, 2, \ldots)$$

where the parameter λ is unknown. Considering n independent observations of X, estimate λ by the method of maximum likelihood.

16. State and prove Blackwell's theorem.

17. The expected value m of a population, where the characteristic x has the normal distribution $N(m\,;1)$ is unknown. Test the hypothesis $H_0(m = m_0)$ against the alternate hypothesis $H_1(m = m_1)$, where $m_1 > m_0$, by applying a most powerful test based on a simple sample for a give α and β. What value should n have?

18. Derive the OC function $L(Q)$ of the Sequential Probability Ratio Test.

19. With the usual notations, prove that $E_{Q'}(n) \cong -\dfrac{\log A \log B}{E_{Q'}(z^2)}$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Define Student's distribution. Also find its density.

21. Suppose that the theoretical frequencies π_k are give. Prove that the sequence $\{F_n(z)\}$ of distribution functions of the statistic X^2 satisfies the relation

$$\lim_{n\to\infty} F_n(z) = \begin{cases} \dfrac{1}{2^{(r-1)/2}\Gamma\left(\frac{1}{2}(r-1)\right)} \displaystyle\int_0^z z^{(r-3)/2} e^{-z/2} dz, & for\ z > 0 \\ 0 & for\ z \le 0 \end{cases}$$

22. State and prove Rao-Cramer inequality.

23. State and prove Fisher's lemma.

24. If the variance of z is different from zero, then prove that the probability that $n = \infty$ is equal to zero.
