DECEMBER 2023 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Define student's non–central t statistic.

2. Define statistic.

3. Define non-parametric hypothesis and non-parametric tests.

4. State Kolmogorov theorem.

5. Define unbiased estimate.

6. Define likelihood function.

7. Define null hypothesis and alternate hypothesis.

8. State Blackwell's theorem.

9. State Neyman's theorem.

10. What is uniformly most powerful test?

11. With the usual notations, prove that $L(Q) = \dfrac{E_Q^{**} - 1}{E_Q^{**} - E_Q^{*}}$.

12. What is sequential analysis?

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. Prove that the sequence $\{F_n(t)\}$ of distribution functions of Student's t with n degrees of freedom satisfies for every t, the relation

$$\lim_{n \to \infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt .$$

14. Derive the distribution of the arithmetic mean of independent normally distributed random variables.

15. We have good and defective items in a lot and the proportion p of the defective items is unknown. From this lot, draw a simple sample of size $n = 30$. Assign the number one to the appearance of a defective item and the number zero to the appearance of a good item. Suppose that there are four defective items in the sample. Test the hypothesis $H_0(p = 0.10)$. Should we reject H_0 at the significance level $\alpha = 0.05$?

16. Consider a population in which the characteristic X has an arbitrary distribution whose first moment exists. Estimate the unknown expected value $E(X) = m$.

17. The characteristic X of elements of a population has the normal distribution $N(m; \sigma)$, with unknown σ and m known. Consider the statistic $U = \frac{1}{n}\sqrt{0.5\pi}\sum_{k=1}^{n}|X_k - m|$, where the random variables X_k are independent and have the same distribution as X. Prove that U is an unbiased estimate of σ. Also find the efficiency of U.

18. Explain uniformly most powerful test.

19. With the usual notations, prove that $E_{Q'}(n) \cong -\frac{\log A \log B}{E_{Q'}(z^2)}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

20. Derive Fisher's Z-distribution.

21. Suppose that the theoretical frequencies π_k are given. Prove that the sequence $\{F_n(z)\}$ of distribution functions of the statistic χ^2 satisfies the relation

$$\lim_{n \to \infty} F_n(z) =$$

$$\begin{cases} \dfrac{1}{2^{(r-1)/2}\ \Gamma\left(\dfrac{1}{2}(r-1)\right)} \int_0^z z^{(r-3)/2}\ e^{-z/2}\ dz & \text{for } z > 0 \\ 0 & \text{for } z \le 0 \end{cases}$$

22. Prove that all unbiased estimate U of the parameter Q is the most efficient if and only if

    (a) the estimate U is sufficient

    (b) for $g(u,Q) > 0$, the density $q(u,Q)$ almost everywhere satisfies the relation

    $\dfrac{\partial \log g(u,Q)}{\partial Q} = c(u - Q)$, where the number c is independent of u.

23. If the point ζ in sample space is a random variable of continuous type with density $f(\zeta, Q)$, where the parameter Q is unknown and $H_0(Q = Q_0)$ and $H_1(Q = Q_1), (Q_0 \neq Q_1)$ are the null and alternative hypotheses, respectively, then prove that a most powerful test-exists.

24. State and prove the fundamental identity.
