DECEMBER 2024 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define statistic.

2. Define Student's noncentral t-statistic.

3. State Kolmogorov theorem.

4. Define parametric hypothesis and parametric tests.

5. Define likelihood function.

6. Define unbiased estimate.

7. State Blackwell's theorem.

8. Define admissible hypothesis.

9. What is uniformly most powerful test?

10. What is sequential analysis

11. Write Wald's fundamental identify.

12. Define composite hypothesis.

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. From a population in which the characteristic X has the same normal distribution N(1, 2), we draw a simple sample of size $n=12$. Observe the following values of X:

$x_1=2.0\, x_2=2.5\, x_3=0.5\, x_4=1.0\, x_5=0.0\, x_6=-0.9$

$x_7=.15\, x_8=-1.5\, x_9=0.8\, x_{10}=1.1\, x_{11}=0.8\ x_{12}=0.4$

Find $\bar{x}$ and s^2. What is the probability that Z will exceed or equal to $z=32.28$?

14. Suppose a simple sample $n=150$ elements is given. The mean and the standard deviation of the sample are, respectively, $\bar{x}=0.4$ and $s=4$. The expected value and the standard deviation of the population are not known. Should we reject the hypothesis $H_0\,(m=0)$?

15. Derive the distribution of the arithmetic mean of independent normally distributed random variables.

16. The characteristic X of elements of a population has the normal distribution $N(m:\sigma)$, with unknown σ and m known. Consider the statistic

$$U=\frac{1}{n}\sqrt{0.5\pi}\sum_{k=1}^{n}|X_k-m|,$$

where the random variables X_k are independent and have the same distribution as X. Prove that U is an unbiased estimate of σ. Also find the efficiency of U.

17. Explain independence tests by contingency tables.

18. If $E(Z)\neq 0$, prove that there exists one and only one real number $h_0\neq 0$ such that $E\left(e^{h_0 z}\right)=1$;

and if $E(z)=0$, then prove that the above equality holds only for $h_0=0$.

19. Explain uniformly most powerful test.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Derive Fisher's Z-distribution.

21. State and prove Smirnov theorem.

22. State and prove Rao-Cramer inequality.

23. If the point ζ in sample space is a random variable of continuous type with density $f(\zeta, Q)$, where the parameter Q is unknown and $H_0(Q=Q_0)$ and $H_1(Q=Q_1)$, $(Q_0 \neq Q_1)$ are the null and alternative hypotheses, respectively, then prove that a most powerful test exists.

24. State and prove Blackwell's theorem.

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