MAY 2022 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define χ^2-statistic.

2. Define the degrees of freedom of z statistic.

3. Define nonparametric hypothesis.

4. What is a theoretical frequency?

5. Define consistent estimate.

6. Define likelihood function of the purely continuous Markov process.

7. State Fisher's lemma.

8. Define an asymptotically unbiased estimate.

9. Write the fundamental identity.

10. Define simple hypothesis.

11. What is sequential analysis?

12. Define the OC function of the Sequential Probability Ratio Test.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. The random variables $X_k(k = 1,2,...,16)$ are independent and have the same density

$$f(x) = \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{1}{2} . \frac{(x-1)^2}{4} \right].$$

Find the density of

$$\overline{X} = \frac{1}{16} \sum_{k=1}^{16} X_k .$$

Also find $P(0 \le \overline{X} \le 2)$.

14. We have good and defective items in a lot and the proportion p of the defective items is unknown. From this lot, draw a simple sample of size n = 30. Assign the number one to the appearance of a defective item and the number zero to the appearance of a good item. Suppose that there are four defective items in the sample. Test the hypothesis H_0 (p = 0.10). Should we reject H_0 at the significance level α = 0.05?

15. Find the maximum likelihood estimates for m and σ^2 for

$$f(\tau,x,t,y,m,\sigma)=\frac{1}{\sigma\sqrt{2\pi(t-\tau)}}\exp\left\{-\frac{[y-x-m(t-\tau)]^2}{2\sigma^2(t-\tau)}\right\}$$

16. The characteristic X of elements of a population has the normal distribution $N(m;\sigma)$, with unknown σ and m known. Consider the statistic

$$U=\frac{1}{n}\sqrt{0.5\pi}\sum_{k=1}^{n}|X_k-m|,$$

where the random variables X_k are independent and have the same distribution as X. Prove that U is an unbiased estimate of σ. Also find the efficiency of U.

17. The expected value m of a population, where the characteristic x has the normal distribution N(m; 1), is unknown. Test the hypothesis $H_0(m = m_0)$ against the alternate hypothesis $H_1(m = m_1)$, where $m_1 > m_0$, by applying a most powerful test based on a simple sample for a given α and β. What value should n have?

18. With the usual notations, prove that $L(Q') \cong \frac{\log A}{\log A - \log B}$.

19. Write a short notes on one way classification.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Derive Fisher's Z-distribution.

21. State and prove Kolmogorov theorem.

22. State and prove Blackwell's theorem.

23. If the point ζ in sample space is a random variable of continuous type with density $f(\zeta, Q)$, where the parameter Q is unknown and $H_0(Q = Q_0)$ and $H_1(Q = Q_1), (Q_0 \neq Q_1)$ are the null and alternative hypotheses, respectively, then prove that a most powerful test exists.

24. If the variance of z is different from zero, then prove that the probability that $n = \infty$ is equal to zero.

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