Time : Three hours Maximum : 80 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Define Student's noncentral t-statistic.

2. Define simple random sample.

3. Define parametric hypothesis and parametric tests.

4. State Kolmogorov theorem.

5. State Cramer's theorem.

6. Define Pearson's χ^2-statistic.

7. Define admissible hypothesis.

8. Define consistent estimate.

9. State Neyman's theorem.

10. Define composite hypothesis.

11. Write the fundamental identity.

12. What is sequential analysis.

SECTION B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions

13. From a population in which the characteristic X has the same normal distribution $N(1,2)$, we draw a simple sample of size $n = 12$. Observe the following values of X:

$$x_1 = 2.0 \;\; x_2 = 2.5 \;\; x_3 = 0.5 \;\; x_4 = 1.0 \;\; x_5 = 0.0 \;\; x_6 = -0.9$$
$$x_7 = .15 \;\; x_8 = -1.5 \; x_9 = 0.8 \;\; x_{10} = 1.1 \; x_{11} = 0.8 \; x_{12} = 0.4$$

Find $\bar{x}$ and s^2. What is the probability that Z will exceed or equal to $z = 32.28$?

14. The random variables $X_k (k = 1,2,...,16)$ are independent and have the same density

$$f(x) = \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{1}{2} \cdot \frac{(x-1)^2}{4} \right].$$

Find the density of $\overline{X} = \frac{1}{6} \sum_{k=1}^{16} X_k$.

Also find $P(0 \le \overline{X} \le 2)$.

15. Consider a simple sample of n = 50 elements drawn from a population in which the characteristic X has the distribution with standard deviation σ=5 and an unknown expected value m. Also $\overline{x} = 2$. Test the hypothesis $H_0(m = 0)$.

16. Explain independence tests by contingency tables.

17. The characteristic X of elements of a population has the Cauchy distribution given by the density

$$f(x) = \frac{1}{\pi[1 + (x - Q)^2]}.$$

where Q is an unknown parameter. Find the minimal variance of a regular and unbiased estimate U of Q.

18. The expected value m of a population, where the characteristic x has the normal distribution $N(m;1)$, is unknown. Test the hypothesis $H_0(m = m_0)$ against the alternate hypothesis $H_1(m = m_1)$, where $m_1 > m_0$, by applying a most powerful test based on a simple sample for a given α and β. What value should n have?

19. If $E(z) \neq 0$, prove that there exists one and only one real number $h_0 \neq 0$ such that $E\left(e^{h_0 z}\right) = 1$; and if $E(z) = 0$, then prove that the above equality holds only for $h_0 = 0$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Find the distribution of the random variable $(\overline{X}, S)$.

21. State and prove Smirnov theorem.

22. State and prove Rao–Cramer inequality.

23. State and prove Fisher's lemma.

24. If the variance of z is different from zero, then prove that the probability that $n = \infty$ is equal to zero.

———————