MAY 2024 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define χ^2 – Statistic.

2. What is random sample?

3. Define parametric hypothesis and parametric tests.

4. Define Snedecor's non-central F.

5. Define consistent estimate.

6. Define likelihood function.

7. Define the likelihood function of the discrete Markov process.

8. Define unbiased estimate.

9. What is uniformly most powerful test?

10. Define the operating characteristic function.

11. Define a biased test.

12. What is sequential analysis?

PART B — ($5 \times 6 = 30$ marks)

Answer any FIVE questions.

13. If $X_k, (k = 1,2,\ldots,j)$, are identically distributed and independent binomial random variables, find the distribution of $\overline{X} = \frac{1}{j}\sum_{k=1}^{j} X_k$.

14. Select two independent simple samples from a population in which the characteristic X has a continuous distribution function. The number of elements of the first sample is $n_1 = 70$, of the second sample $n_2 = 100$. The observed value of $D_{n1\,n2}$ is 0.18. What is the probability that $D_{n1\,n2}$ is smaller than this value?

15. Suppose a simple sample $n = 150$ elements is given. The mean and the standard deviation of the sample are, respectively, $\overline{x} = 0.4$ and $s = 4$. The expected value and the standard deviation of the population are not known. Should we reject the hypothesis $H_0(m = 0)$?

16. Explain independence tests by contingency tables.

17. The random variable X has the Poisson distribution.

$$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda},(k=0,1,2,....)$$

where the parameter λ is unknown, Considering n independent observations of X, estimate λ by the method of maximum likelihood. Find the most efficient unbiased estimate of λ.

18. Explain uniformly most powerful test.

19. With the usual notation, prove that $L(Q')\cong\frac{\log A}{\log A-\log B}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the distribution of the random variable $(\overline{X},S)$.

21. State and prove Smirnov theorem.

22. Prove that an unbiased estimate U of the parameter Q is the most efficient if and only if

(i) the estimate U is sufficient

(ii) for $g(u,Q)>0$, the density $g(u,Q)$ almost everywhere satisfies the relation $\frac{\partial \log g(u,Q)}{\partial Q}=c(u-Q)$,

where the number c is independent of u.

23. State and prove Blackwell's theorem.

24. If the point ζ in sample is a random variable of continuous type with density $f(\zeta, Q)$, where the parameter Q is unknown and $H_0(Q = Q_0)$ and $H_1(Q = Q_1), (Q_0 \neq Q_1)$ are the null and alternative hypotheses, respectively, then prove that a most powerful test exists.

———————