MAY 2025 **P/ID 37520/PMA4W**

Time : Three hours Maximum : 80 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define simple random sample.

2. Define student's non-central t statistic.

3. State Kolmogorov theorem.

4. Define parametric hypothesis and parametric tests.

5. Define Pearson's χ^2 -statistic.

6. Define consistent estimate.

7. Define unbiased estimate.

8. Define null hypothesis and alternate hypothesis.

9. State Neyman's theorem.

10. Define a biased test.

11. Write the fundamental identity.

12. What is uniformly most powerful test?

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Prove that the sequence $\{F_n(t)\}$ of distribution functions of Student's t with n degrees of freedom satisfies for every t, the relation

$$\lim_{n\to\infty} F(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt$$

14. Select two independent simple samples from a population in which the characteristic X has a continuous distribution function. The number of elements of the first sample is $n_1 = 70$, of the second sample $n_2 = 100$. The observed value of $D_{n_1 n_2}$ is 0.18. What is the probability that $D_{n_1 n_2}$ is smaller than this value?

15. Consider a population in which the characteristic X has an arbitrary distribution whose first moment exists. Estimate the unknown expected value $E(X) = m$.

16. Consider a simple sample of $n = 50$ elements drawn from a population in which the characteristic X has the distribution with standard deviation $\sigma = 5$ and an unknown expected value m. Also $\bar{x} = 2$. Test the hypothesis $H_0(m = 0)$.

17. The random variable X has the Poisson distribution

$$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda},\ (k=0,1,2,...)$$

where the parameter λ is unknown. Considering n independent observations of X, estimate λ by the method of maximum likelihood. Find the most efficient unbiased estimate of λ.

18. With the usual notations, prove that $E_{Q'}(n)\cong-\frac{\log A\log B}{E_{Q'}(z^2)}$.

19. The expected value m of a population, where the characteristic x has the normal distribution $N(m;1)$, is unknown. Test the hypothesis $H_0(m=m_0)$ against the alternate hypothesis $H_1(m=m_1)$, where $m_1>m_0$, by applying a most powerful test based on a simple sample for a given α and β. What value should n have?

PART C — ($3\times10=30$ marks)

Answer any THREE questions.

20. Find the distribution of the random variable $(\overline{X},S)$.

21. State and prove Smirnov theorem.

22. Prove that an unbiased estimate U of the parameter Q is the most efficient if and only if

(a) the estimate U is sufficient

(b) for $g(u, Q) > 0$, the density $g(u,Q)$ almost everywhere satisfies the relation

$$\frac{\partial \log g(u,Q)}{\partial Q} = c(u - Q),$$

where the number c is independent of u.

23. State and prove Blackwell's theorem.

24. If the variance of z is different from zero, then prove that the probability that $n = \infty$ is equal to zero.
