

MATHEMATICS

1. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C) =$
- 1) $\{(a, c), (a, d), (b, d)\}$ 2) $\{(c, a), (d, a)\}$
 3) $\{(a, b), (c, d)\}$ 4) $\{(a, c), (a, d)\}$
2. The function $f: X \rightarrow Y$ defined by $f(x) = \sin x$ is one-one but not onto if X and Y are respectively equal to,
- 1) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$ 2) $\left[0, \frac{\pi}{2}\right]$ and $[-1, 1]$
 3) $[0, \pi]$ and $[0, 1]$ 4) \mathbb{R} and \mathbb{R}
3. If $\log_4^2 + \log_4^4 + \log_4^{16} + \log_4^x = 6$, then $x =$
- 1) 32 2) 8
 3) 4 4) 64
4. If $S_n = \frac{1}{6.11} + \frac{1}{11.16} + \frac{1}{16.21} + \dots$ to n terms, then $6S_n =$
- 1) $\frac{1}{(5n+6)}$ 2) $\frac{(2n-1)}{5n+6}$
 3) $\frac{n}{(5n+6)}$ 4) $\frac{5n-4}{5n+6}$
5. The remainder obtained when $(1)^2 + (2)^2 + (3)^2 + \dots + (100)^2$ is divided by 10^2 is
- 1) 14 2) 17
 3) 28 4) 27

(Space for Rough Work)

6. If $(p \wedge \sim r) \rightarrow (\sim p \vee q)$ is false, then the truth values of p, q and r are respectively
- | | |
|-------------------|-------------------|
| 1) T, F and T | 2) F, T and T |
| 3) F, F and T | 4) T, F and F |
7. If α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then $\sum \alpha^2$ and $\sum \frac{1}{\alpha\beta}$ are respectively =
- | | |
|-------------|--------------|
| 1) 16 and 0 | 2) -16 and 0 |
| 3) 16 and 8 | 4) 0 and -16 |
8. The g.c.d. of 1080 and 675 is
- | | |
|--------|--------|
| 1) 125 | 2) 225 |
| 3) 135 | 4) 145 |
9. If $a \mid (b+c)$ and $a \mid (b-c)$ where $a, b, c \in \mathbb{N}$ then,
- | | |
|--------------------------------|--------------------------------|
| 1) $c^2 \equiv a^2 \pmod{b^2}$ | 2) $a^2 \equiv b^2 \pmod{c^2}$ |
| 3) $a^2 + c^2 = b^2$ | 4) $b^2 \equiv c^2 \pmod{a^2}$ |
10. If a, b and $c \in \mathbb{N}$ which one of the following is not true ?
- | | |
|---|---|
| 1) $a \mid b$ and $a \mid c \Rightarrow a \mid b+c$ | 2) $a \mid b+c \Rightarrow a \mid b$ and $a \mid c$ |
| 3) $a \mid b$ and $b \mid c \Rightarrow a \mid c$ | 4) $a \mid b$ and $a \mid c \Rightarrow a \mid 3b+2c$ |

(Space for Rough Work)

11. If $2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$, then $B =$

1) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$

3) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

4) $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

12. If $O(A) = 2 \times 3$, $O(B) = 3 \times 2$, and $O(C) = 3 \times 3$, which one of the following is not defined?

1) $C(A+B')$

2) $C(A+B')$

3) BAC

4) $CB+A'$

13. If $A = \begin{bmatrix} 1 & -3 \\ 2 & K \end{bmatrix}$ and $A^2 - 4A + 10I = A$, then $K =$

1) 1 or 4

2) 4 and not 1

3) -4

4) 0

14. The value of $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix} =$

1) 0

2) $(x+y+z)^3$

3) $2(x+y+z)^3$

4) $2(x+y+z)^2$

15. On the set Q of all rational numbers the operation $*$ which is both associative and commutative is given by $a * b =$

1) $2a + 3b$

2) $ab + 1$

3) $a^2 + b^2$

4) $a + b + ab$

(Space for Rough Work)

26. If the length of the tangent from any point on the circle $(x-3)^2 + (y+2)^2 = 5r^2$ to the circle $(x-3)^2 + (y+2)^2 = r^2$ is 16 units, then the area between the two circles in sq. units is
- 1) 16π
 - 2) 8π
 - 3) 4π
 - 4) 32π
27. The circles $ax^2 + ay^2 + 2g_1x + 2f_1y + c_1 = 0$ and $bx^2 + by^2 + 2g_2x + 2f_2y + c_2 = 0$ ($a \neq 0$ and $b \neq 0$) cut orthogonally if
- 1) $g_1g_2 + f_1f_2 = c_1 + c_2$
 - 2) $bg_1g_2 + af_1f_2 = bc_1 + ac_2$
 - 3) $g_1g_2 + f_1f_2 = bc_1 + ac_2$
 - 4) $g_1g_2 + f_1f_2 = ac_1 + bc_2$
28. The equation of the common tangent of the two touching circles, $y^2 + x^2 - 6x - 12y + 37 = 0$ and $x^2 + y^2 - 6y + 7 = 0$ is
- 1) $x + y + 5 = 0$
 - 2) $x + y - 5 = 0$
 - 3) $x - y + 5 = 0$
 - 4) $x - y - 5 = 0$
29. The equation of the parabola with vertex at $(-1, 1)$ and focus $(2, 1)$ is
- 1) $y^2 - 2y - 12x + 13 = 0$
 - 2) $y^2 - 2y + 12x + 11 = 0$
 - 3) $x^2 + 2x - 12y + 13 = 0$
 - 4) $y^2 - 2y - 12x - 11 = 0$
30. The equation of the line which is tangent to both the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 40x$ is
- 1) $2x + y + 5 = 0$
 - 2) $2x - y - 5 = 0$
 - 3) $2x - y + 5 = 0$
 - 4) $2x - y \pm 5 = 0$

(Space for Rough Work)

31. $x = 4(1 + \cos \theta)$ and $y = 3(1 + \sin \theta)$ are the parametric equations of

1) $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

2) $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$

3) $\frac{(x+4)^2}{16} + \frac{(y+3)^2}{9} = 1$

4) $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$

32. If the distance between the foci and the distance between the directrices of the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3 : 2, then $a : b$ is =

1) 2 : 1

2) 1 : 2

3) $\sqrt{3} : \sqrt{2}$

4) $\sqrt{2} : 1$

33. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common

1) centre and vertices only

2) centre, foci and vertices

3) centre, foci and directrices

4) centre only

34. If $\sec \theta = m$ and $\tan \theta = n$, then $\frac{1}{m} \left[(m+n) + \frac{1}{(m+n)} \right] =$

1) mn

2) $2n$

3) $2m$

4) 2

35. The value of $\frac{\sin 85^\circ - \sin 15^\circ}{\cos 65^\circ} =$

1) 0

2) 1

3) -1

4) 2

(Space for Rough Work)

36. From an aeroplane flying, vertically above a horizontal road, the angles of depression of two consecutive stones on the same side of the aeroplane are observed to be 30° and 60° respectively. The height at which the aeroplane is flying in km is

- 1) 2
2) $\frac{2}{\sqrt{3}}$
3) $\frac{\sqrt{3}}{2}$
4) $\frac{4}{\sqrt{3}}$

37. If the angles of a triangle are in the ratio 3 : 4 : 5, then the sides are in the ratio

- 1) 3 : 4 : 5
2) $2 : \sqrt{3} : \sqrt{3} + 1$
3) $\sqrt{2} : \sqrt{6} : \sqrt{3} + 1$
4) $2 : \sqrt{6} : \sqrt{3} + 1$

38. If $\cos^{-1}x = \alpha$, ($0 < x < 1$) and $\sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$,

then $\tan^{-1}(2x) =$

- 1) $\frac{\pi}{2}$
2) $\frac{\pi}{3}$
3) $\frac{\pi}{4}$
4) $\frac{\pi}{6}$

39. If $a > b > 0$, then the value of $\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$ depends on

- 1) neither a nor b
2) a and not b
3) b and not a
4) both a and b

40. Which one of the following equations has no solution ?

- 1) $\sqrt{3} \sin \theta - \cos \theta = 2$
2) $\cos \theta + \sin \theta = \sqrt{2}$
3) $\operatorname{cosec} \theta \cdot \sec \theta = 1$
4) $\operatorname{cosec} \theta - \sec \theta = \operatorname{cosec} \theta \cdot \sec \theta$

(Space for Rough Work)

41. The complex number $\frac{(-\sqrt{3} + 3i)(1-i)}{(3 + \sqrt{3}i)(i)(\sqrt{3} + \sqrt{3}i)}$ when represented in the Argand diagram lies
- 1) on the X-axis (Real axis)
 - 2) on the Y-axis (Imaginary axis)
 - 3) in the first quadrant
 - 4) in the second quadrant
42. If $2x = -1 + \sqrt{3}i$, then the value of $(1 - x^2 + x)^6 - (1 - x + x^2)^6 =$
- 1) 0
 - 2) 64
 - 3) -64
 - 4) 32
43. The modulus and amplitude of $(1 + i\sqrt{3})^8$ are respectively
- 1) 256 and $\frac{8\pi}{3}$
 - 2) 2 and $\frac{2\pi}{3}$
 - 3) 256 and $\frac{2\pi}{3}$
 - 4) 256 and $\frac{\pi}{3}$
44. The value of $\lim_{x \rightarrow 0} \frac{5^x - 5^{-x}}{2x} =$
- 1) $2 \log 5$
 - 2) 1
 - 3) 0
 - 4) $\log 5$
45. Which one of the following is not true always?
- 1) If a function $f(x)$ is continuous at $x = a$, then $\lim_{x \rightarrow a} f(x)$ exists.
 - 2) If $f(x)$ and $g(x)$ are differentiable at $x = a$, then $f(x) + g(x)$ is also differentiable at $x = a$.
 - 3) If $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$.
 - 4) If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$.

(Space for Rough Work)

46. If $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots$ to ∞ with $|x| > 1$ then $\frac{dy}{dx} =$

1) $\frac{-y^2}{x^2}$

2) $\frac{y^2}{x^2}$

3) $x^2 y^2$

4) $\frac{x^2}{y^2}$

47. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and $f \circ g(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$

1) $3x^2 + \frac{3}{x^4}$

2) $1 + \frac{1}{x^2}$

3) $x^2 - \frac{1}{x^2}$

4) $3x^2 - 3$

48. The derivative of $a^{\sec x}$ w.r.t. $a^{\tan x}$ ($a > 0$) is

1) $a^{\sec x - \tan x}$

2) $\sin x a^{\sec x - \tan x}$

3) $\sin x a^{\tan x - \sec x}$

4) $\sec x a^{\sec x - \tan x}$

49. If $\sin(x+y) + \cos(x+y) = \log(x+y)$, then $\frac{d^2y}{dx^2} =$

1) 1

2) -1

3) 0

4) $-\frac{y}{x}$

50. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) = 8$, then $g(8) =$

1) 8

2) 3

3) 0

4) 5

(Space for Rough Work)

56. $\int \frac{dx}{x\sqrt{x^6-16}} =$

1) $\text{Sec}^{-1}\left(\frac{x^3}{4}\right) + c$

2) $\frac{1}{12} \text{Sec}^{-1}\left(\frac{x^3}{4}\right) + c$

3) $\text{Cosh}^{-1}\left(\frac{x^3}{4}\right) + c$

4) $\frac{1}{3} \text{Sec}^{-1}\left(\frac{x^3}{4}\right) + c$

57. If $I_1 = \int_0^{\pi/2} x \sin x \, dx$ and $I_2 = \int_0^{\pi/2} x \cos x \, dx$, then which one of the following is true ?

1) $I_1 = I_2$

2) $I_1 + I_2 = 0$

3) $I_1 = \frac{\pi}{2} I_2$

4) $I_1 + I_2 = \frac{\pi}{2}$

58. If $f(x)$ is defined in $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{(x^2 + 3)}$, then

$$\int_{-2}^2 g(x) dx =$$

1) 24

2) 0

3) -48

4) 64

59. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in sq. units =

1) $\frac{4}{3}$

2) $\frac{2}{3}$

3) $\frac{1}{3}$

4) $\frac{8}{3}$

60. The solution of the differential equation $e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)y(dx) = 0$ subjected to the condition that $y = 1$ when $x = 0$ is

1) $(y+1) + e^x \cos^2 x = 2$

2) $y + \text{Log } y = e^x \cos^2 x$

3) $\text{Log}(y+1) + e^x \cos^2 x = 1$

4) $y + \text{Log } y + e^x \cos^2 x = 2$

(Space for Rough Work)