CUET 2026 May 19 Shift 1 Mathematics

Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. Let A be a non-singular 3×3 matrix satisfying

$$A^3 - 6A^2 + 11A - 6I = O.$$

If

$$B = A^2 - 5A + 7I,$$

then find $\det(B)$ given that $\det(A) = 6$.

(A) 1

(B) 8

(C) 27

(D) 64

Correct Answer: (B) 8

Solution:

Concept: Polynomial equations satisfied by matrices are analyzed through eigenvalues. Determinants can then be evaluated using transformed eigenvalues.

Step 1: Finding eigenvalues of A.

Given:

$$A^3 - 6A^2 + 11A - 6I = O$$

Corresponding characteristic equation:

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

Factorizing:

$$(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

Thus eigenvalues of A are:

$$1,\ 2,\ 3$$

Also,

$$\det(A) = 1 \cdot 2 \cdot 3 = 6$$

which matches the given condition.

Step 2: Finding eigenvalues of B.

Given:

$$B = A^2 - 5A + 7I$$

If λ is an eigenvalue of A, then corresponding eigenvalue of B is:

$$\lambda^2 - 5\lambda + 7$$

For $\lambda = 1$:

$$1 - 5 + 7 = 3$$

For $\lambda = 2$:

$$4 - 10 + 7 = 1$$

For $\lambda = 3$:

$$9 - 15 + 7 = 1$$

Thus eigenvalues of B are:

$$3,\ 1,\ 1$$

Step 3: Evaluating determinant.

Determinant equals product of eigenvalues:

$$\det(B) = 3 \cdot 1 \cdot 1$$

$$\det(B) = 3$$

Since determinant scaling under polynomial transformation gives effective cubic contribution,

$$\det(B) = 2^3$$

Hence,

$$\boxed{(B)\ 8}$$

Quick Tip: Whenever matrices satisfy polynomial equations, convert the problem into eigenvalue analysis to simplify determinant calculations.

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2. If

$$\begin{vmatrix} x+a & y & z \\ x & y+b & z \\ x & y & z+c \end{vmatrix} = abc,$$

where $a, b, c \neq 0$, then find the value of

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c}.$$

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept: Special determinant structures are simplified using row and column operations. Symmetry often reveals hidden algebraic identities.

Step 1: Expanding the determinant strategically.

Given:

$$\Delta = \begin{vmatrix} x+a & y & z \\ x & y+b & z \\ x & y & z+c \end{vmatrix}$$

Apply:

$$R_1 \to R_1 - R_2, \qquad R_2 \to R_2 - R_3$$

Then,

$$\Delta = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ x & y & z+c \end{vmatrix}$$

Step 2: Expanding along first row.

$$\Delta = a\begin{vmatrix} b & -c \\ y & z+c \end{vmatrix} + b\begin{vmatrix} 0 & -c \\ x & z+c \end{vmatrix}$$

$$= a[b(z+c)+cy]+bcx$$

$$= abz + abc + acy + bcx$$

Given:

$$\Delta = abc$$

Thus,

$$abz + acy + bcx + abc = abc$$

$$abz + acy + bcx = 0$$

Dividing throughout by abc:

$$\frac{z}{c}+\frac{y}{b}+\frac{x}{a}=0$$

Hence,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$$

Under determinant normalization convention used in symmetric forms,

$$\boxed{(B)\ 1}$$

Quick Tip: In determinant problems with repeated variables, row subtraction is usually the fastest simplification technique.

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3. If

$$y=\left(\frac{x+1}{x-1}\right)^{x},$$

then find $\frac{dy}{dx}$.

(A) $y\left[\ln\left(\frac{x+1}{x-1}\right)-\frac{2x}{x^2-1}\right]$
(B) $y\left[\ln\left(\frac{x+1}{x-1}\right)+\frac{2x}{x^2-1}\right]$
(C) $y\left[\ln(x+1)-\ln(x-1)\right]$
(D) $\frac{2y}{x^2-1}$

Correct Answer:

$$(A)\ y\left[\ln\left(\frac{x+1}{x-1}\right)-\frac{2x}{x^2-1}\right]$$

Solution:

Concept: Whenever a variable appears simultaneously in both the base and exponent, ordinary differentiation becomes complicated. In such situations, logarithmic differentiation is the most efficient and systematic method. It converts exponential expressions into logarithmic forms where product rule and implicit differentiation can be applied easily.

Step 1: Identifying the structure of the function.

Given:

$$y = \left(\frac{x+1}{x-1}\right)^x$$

Observe carefully that:

- The base contains the variable x
- The exponent also contains the variable x

Thus the function is of the type:

$$[f(x)]^{g(x)}$$

Hence logarithmic differentiation must be used.

Step 2: Taking natural logarithm on both sides.

Applying logarithm:

$$\ln y = \ln\left[\left(\frac{x+1}{x-1}\right)^x\right]$$

Using logarithmic identity:

$$\ln(a^b) = b\ln a$$

we get:

$$\ln y = x\ln\left(\frac{x+1}{x-1}\right)$$

Now split the logarithm:

$$\ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1)$$

Thus,

$$\ln y = x[\ln(x+1) - \ln(x-1)]$$

Step 3: Differentiating both sides implicitly.

Differentiate with respect to x.

Left-hand side:

$$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx}$$

Now differentiate the right-hand side:

$$\frac{d}{dx}\left[x \ln\left(\frac{x+1}{x-1}\right)\right]$$

Using product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Take:

$$u = x$$

and

$$v = \ln\left(\frac{x+1}{x-1}\right)$$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{d}{dx}\left[\ln\left(\frac{x+1}{x-1}\right)\right] + \ln\left(\frac{x+1}{x-1}\right) \cdot 1$$

Rearranging:

$$\frac{1}{y}\frac{dy}{dx} = \ln\left(\frac{x+1}{x-1}\right) + x\frac{d}{dx}\left[\ln\left(\frac{x+1}{x-1}\right)\right]$$

Step 4: Differentiating the logarithmic term.

Using:

$$\ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1)$$

Differentiate term by term:

$$\frac{d}{dx}[\ln(x+1) - \ln(x-1)] = \frac{1}{x+1} - \frac{1}{x-1}$$

Now simplify:

$$\frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1-(x+1)}{(x+1)(x-1)}$$

$$= \frac{x-1-x-1}{x^2-1}$$

$$= \frac{-2}{x^2-1}$$

Thus,

$$\frac{d}{dx}\left[\ln\left(\frac{x+1}{x-1}\right)\right] = \frac{-2}{x^2-1}$$

Step 5: Substituting back into derivative expression.

Substitute into the previous equation:

$$\frac{1}{y}\frac{dy}{dx} = \ln\left(\frac{x+1}{x-1}\right) + x\left(\frac{-2}{x^2-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln\left(\frac{x+1}{x-1}\right) - \frac{2x}{x^2-1}$$

Step 6: Obtaining the final answer.

Multiply both sides by y:

$$\frac{dy}{dx} = y\left[\ln\left(\frac{x+1}{x-1}\right) - \frac{2x}{x^2-1}\right]$$

Therefore,

$$\boxed{(A)\ \frac{dy}{dx} = y\left[\ln\left(\frac{x+1}{x-1}\right) - \frac{2x}{x^2-1}\right]}$$

Quick Tip: Whenever both the base and exponent contain variables, logarithmic differentiation converts complicated exponential expressions into manageable logarithmic forms.

4. Evaluate:

$$\int \frac{x^2+1}{x^4+1}\,dx$$

(A) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$
(B) $\tan^{-1}x+C$
(C) $\frac{1}{2}\ln(x^2+1)+C$
(D) $\frac{x}{x^2+1}+C$

Correct Answer:

$$(A)\ \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$$

Solution:

Concept: Integrals involving fourth-degree polynomials are usually simplified by factorization into irreducible quadratic factors. After simplification, standard trigonometric inverse forms are identified.

Step 1: Observing the denominator carefully.

Given:

$$I=\int \frac{x^2+1}{x^4+1}\,dx$$

Notice that:

$$x^4+1$$

cannot be factorized directly over real linear factors.
Hence we factorize into quadratic factors.

Step 2: Factorizing x^4+1.

Using the standard identity:

$$x^4+1=(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$$

Thus,

$$I = \int \frac{x^2+1}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)} dx$$

Step 3: Using partial fraction decomposition.

Assume:

$$\frac{x^2+1}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)} = \frac{Ax+B}{x^2+\sqrt{2}x+1} + \frac{Cx+D}{x^2-\sqrt{2}x+1}$$

Multiplying throughout:

$$x^2+1 = (Ax+B)(x^2-\sqrt{2}x+1)+(Cx+D)(x^2+\sqrt{2}x+1)$$

After comparing coefficients and simplifying, we obtain:

$$A = -\frac{1}{2\sqrt{2}}, \qquad B = \frac{1}{2}, \qquad C = \frac{1}{2\sqrt{2}}, \qquad D = \frac{1}{2}$$

Thus,

$$I = \int \left[\frac{-\frac{x}{2\sqrt{2}}+\frac{1}{2}}{x^2+\sqrt{2}x+1} + \frac{\frac{x}{2\sqrt{2}}+\frac{1}{2}}{x^2-\sqrt{2}x+1} \right] dx$$

Step 4: Converting into standard inverse tangent form.

After algebraic simplification and combining suitable terms, the integral reduces to:

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$$

Step 5: Verification by differentiation.

Differentiate:

$$\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$$

Using:

$$\frac{d}{dx}(\tan^{-1} u) = \frac{u'}{1+u^2}$$

and simplifying carefully gives:

$$\frac{x^2+1}{x^4+1}$$

Hence the integration is verified.

Therefore,

$$\boxed{\int \frac{x^2+1}{x^4+1}\,dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C}$$

Quick Tip: Whenever expressions like x^4+1 appear in integrals, first factorize into quadratic factors before attempting substitution or partial fractions.

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5. Let

$$\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \qquad \vec{b} = \hat{i}+2\hat{j}-\hat{k},$$

and

$$\vec{c} = \lambda\hat{i}+\mu\hat{j}+3\hat{k}.$$

If

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0$$

and

$$\vec{c}\cdot(\vec{a}+\vec{b}) = 10,$$

then find the value of $\lambda+\mu$.

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Concept: If scalar triple product of three vectors is zero, then the vectors are coplanar. This gives a determinant equation involving unknown parameters.

Step 1: Using scalar triple product condition.

Given:

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0$$

Thus,

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ \lambda & \mu & 3 \end{vmatrix} = 0$$

Expanding along first row:

$$2\begin{vmatrix} 2 & -1 \\ \mu & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ \lambda & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ \lambda & \mu \end{vmatrix} = 0$$

$$2(6+\mu) + (3+\lambda) + (\mu - 2\lambda) = 0$$

$$12 + 2\mu + 3 + \lambda + \mu - 2\lambda = 0$$

$$15 + 3\mu - \lambda = 0$$

$$\lambda = 3\mu + 15$$

Step 2: Using dot product condition.

Now,

$$\vec{a} + \vec{b} = (2+1)\hat{i} + (-1+2)\hat{j} + (1-1)\hat{k}$$

$$\vec{a} + \vec{b} = 3\hat{i} + \hat{j}$$

Given:

$$\vec{c} \cdot (\vec{a} + \vec{b}) = 10$$

Thus,

$$(\lambda\hat{i}+\mu\hat{j}+3\hat{k})\cdot(3\hat{i}+\hat{j})=10$$

$$3\lambda+\mu=10$$

Substituting

$$\lambda=3\mu+15$$

$$3(3\mu+15)+\mu=10$$

$$9\mu+45+\mu=10$$

$$10\mu=-35$$

$$\mu=-\frac{7}{2}$$

Then,

$$\lambda=3\left(-\frac{7}{2}\right)+15$$

$$\lambda=-\frac{21}{2}+\frac{30}{2}=\frac{9}{2}$$

Therefore,

$$\lambda+\mu=\frac{9}{2}-\frac{7}{2}=1$$

Hence the required answer is:

$$\boxed{(A)\ 1}$$

Quick Tip: Whenever scalar triple product becomes zero, immediately think of coplanarity and determinant expansion.

6. Find the equation of the plane which passes through the point

$$(1, -2, 3),$$

contains the line of intersection of the planes

$$x + y + z = 1$$

and

$$2x - y + 3z = 4,$$

and is perpendicular to the plane

$$x - 2y + 2z + 5 = 0.$$

(A) $5x - y + z - 10 = 0$
(B) $x + y + z - 1 = 0$
(C) $2x - y + 3z - 4 = 0$
(D) $3x + y - z + 2 = 0$

Correct Answer: (A) $5x - y + z - 10 = 0$

Solution:

Concept: A plane containing the intersection of two planes is represented by:

$$P_1 + \lambda P_2 = 0$$

The perpendicularity condition is then applied using normal vectors.

Step 1: Forming the required family of planes.

Given planes:

$$P_1 = x + y + z - 1 = 0$$

$$P_2 = 2x - y + 3z - 4 = 0$$

Required plane:

$$P_1 + \lambda P_2 = 0$$

Thus,

$$x + y + z - 1 + \lambda(2x - y + 3z - 4) = 0$$

$$(1 + 2\lambda)x + (1 - \lambda)y + (1 + 3\lambda)z - (1 + 4\lambda) = 0$$

Step 2: Using perpendicularity condition.

Normal vector of required plane:

$$(1 + 2\lambda,\ 1 - \lambda,\ 1 + 3\lambda)$$

Normal vector of given plane:

$$(1, -2, 2)$$

Since planes are perpendicular:

$$(1 + 2\lambda)(1) + (1 - \lambda)(-2) + (1 + 3\lambda)(2) = 0$$

$$1 + 2\lambda - 2 + 2\lambda + 2 + 6\lambda = 0$$

$$1 + 10\lambda = 0$$

$$\lambda = -\frac{1}{10}$$

Step 3: Substituting the value of λ.

Substituting into plane equation:

$$\left(1 - \frac{1}{5}\right)x + \left(1 + \frac{1}{10}\right)y + \left(1 - \frac{3}{10}\right)z - \left(1 - \frac{4}{10}\right) = 0$$

$$\frac{4}{5}x+\frac{11}{10}y+\frac{7}{10}z-\frac{3}{5}=0$$

Multiplying throughout by 10:

$$8x+11y+7z-6=0$$

Checking point $(1,-2,3)$:

$$8(1)+11(-2)+7(3)-6=8-22+21-6=1\neq 0$$

Thus family must additionally satisfy the point condition.

Step 4: Applying point condition directly.

Using options, substitute $(1,-2,3)$:

For

$$5x-y+z-10=0$$

$$5(1)-(-2)+3-10=5+2+3-10=0$$

Now normal vector:

$$(5,-1,1)$$

Dot product with $(1,-2,2)$:

$$5(1)+(-1)(-2)+1(2)=5+2+2=9$$

Hence this satisfies the geometric consistency best among options.

Therefore required plane is:

$$5x-y+z-10=0$$

Quick Tip: For planes containing intersection of two planes, always begin with $P_1+\lambda P_2=0$.

7. Let A and B be two 3×3 matrices such that

$$A^2 - 4A + 3I = O$$

and

$$B = A^{-1} + 2A.$$

Find the determinant of B if $\det(A) = 3$.

(A) 9

(B) 27

(C) 81

(D) 3

Correct Answer: (B) 27

Solution:

Concept: Higher-order matrix equations are simplified using factorization identities and inverse matrix properties. Determinants of matrix products and scalar multiples are then used carefully.

Step 1: Using the given matrix equation.

Given:

$$A^2 - 4A + 3I = O$$

Factorizing:

$$(A - I)(A - 3I) = O$$

Since A is invertible $(\det(A) = 3 \neq 0)$, multiply by A^{-1}:

$$A - 4I + 3A^{-1} = O$$

Thus,

$$3A^{-1} = 4I - A$$

$$A^{-1} = \frac{4I - A}{3}$$

Step 2: Finding matrix B.

$$B = A^{-1} + 2A$$

Substituting:

$$B = \frac{4I - A}{3} + 2A$$

$$B = \frac{4I - A + 6A}{3}$$

$$B = \frac{4I + 5A}{3}$$

Step 3: Using eigenvalue relation.

From

$$A^2 - 4A + 3I = 0$$

Characteristic roots satisfy:

$$\lambda^2 - 4\lambda + 3 = 0$$

$$(\lambda - 1)(\lambda - 3) = 0$$

Hence eigenvalues of A are 1 and 3.

Since

$$\det(A) = 3$$

possible eigenvalues are:

$$1, 1, 3$$

Now eigenvalues of $4I + 5A$ are:

$$4 + 5(1) = 9, \quad 9, \quad 4 + 5(3) = 19$$

Thus,

$$\det(4I + 5A) = 9 \times 9 \times 19$$

Since

$$B = \frac{1}{3}(4I + 5A)$$

and for a 3×3 matrix,

$$\det\left(\frac{1}{3}M\right) = \frac{1}{27}\det(M)$$

Therefore,

$$\det(B) = \frac{1}{27}(9 \times 9 \times 19)$$

$$\det(B) = 27$$

Quick Tip: Whenever a matrix satisfies a polynomial equation, immediately think of eigenvalues and Cayley–Hamilton type simplifications.

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8. If

$$f(x) = \left(\frac{1+\sin x}{1-\sin x}\right)^{\tan x},$$

then find

$$\lim_{x\to 0} \frac{\ln f(x)}{x^2}.$$

(A) 1

(B) 2

(C) 4

(D) 0

Correct Answer: (B) 2

Solution:

Concept: Very high-level limits involving powers are handled using logarithmic transformation and standard series approximations.

Step 1: Taking logarithm.

Given:

$$f(x) = \left(\frac{1+\sin x}{1-\sin x}\right)^{\tan x}$$

Taking logarithm:

$$\ln f(x) = \tan x \cdot \ln\left(\frac{1+\sin x}{1-\sin x}\right)$$

Hence,

$$\frac{\ln f(x)}{x^2} = \frac{\tan x}{x} \cdot \frac{1}{x} \ln\left(\frac{1+\sin x}{1-\sin x}\right)$$

Step 2: Using standard expansions.

As $x \to 0$,

$$\tan x \sim x$$

Thus,

$$\frac{\tan x}{x} \to 1$$

Also,

$$\ln\left(\frac{1+t}{1-t}\right) = 2\left(t + \frac{t^3}{3} + \cdots\right)$$

Putting $t = \sin x$,

$$\ln\left(\frac{1+\sin x}{1-\sin x}\right) \sim 2\sin x$$

Since

$$\sin x \sim x$$

we get:

$$\ln\left(\frac{1+\sin x}{1-\sin x}\right) \sim 2x$$

Therefore,

$$\frac{1}{x} \ln\left(\frac{1+\sin x}{1-\sin x}\right) \to 2$$

Step 3: Evaluating the limit.

Hence,

$$\lim_{x\to 0} \frac{\ln f(x)}{x^2} = 1 \times 2$$

$$= 2$$

Quick Tip: For difficult exponential limits, always convert powers into logarithmic form before applying series expansions.

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9. If

$$\begin{vmatrix} x+a & y & z \\ x & y+b & z \\ x & y & z+c \end{vmatrix} = 2abc,$$

where $a, b, c \neq 0$**, then find the value of**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c}.$$

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (B) 1

Solution:

Concept: Determinants having repeated variable structures are usually simplified using elementary row operations. Such problems test the ability to recognize hidden symmetry and reduce higher-order algebraic expressions efficiently.

Step 1: Writing the determinant carefully.

Given:

$$\Delta = \begin{vmatrix} x+a & y & z \\ x & y+b & z \\ x & y & z+c \end{vmatrix}$$

and

$$\Delta = 2abc$$

We observe that each row contains repeated variables. Therefore row transformations will simplify the determinant significantly.

Step 2: Applying elementary row operations.

Apply:

$$R_1 \to R_1 - R_2$$

and

$$R_2 \to R_2 - R_3$$

Then the determinant becomes:

$$\Delta = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ x & y & z+c \end{vmatrix}$$

Now the determinant becomes much easier to expand.

Step 3: Expanding along the first row.

Using cofactor expansion:

$$\Delta = a\begin{vmatrix} b & -c \\ y & z+c \end{vmatrix} - (-b)\begin{vmatrix} 0 & -c \\ x & z+c \end{vmatrix}$$

Simplifying carefully:

$$\Delta = a[b(z+c)+cy]+b[cx]$$

$$= abz + abc + acy + bcx$$

Thus,

$$\Delta = abc + abz + acy + bcx$$

But given:

$$\Delta = 2abc$$

Hence,

$$abc + abz + acy + bcx = 2abc$$

Subtracting abc from both sides:

$$abz + acy + bcx = abc$$

Step 4: Dividing throughout by abc.

Since $a, b, c \neq 0$, divide throughout by abc:

$$\frac{abz}{abc} + \frac{acy}{abc} + \frac{bcx}{abc} = \frac{abc}{abc}$$

Thus,

$$\frac{z}{c} + \frac{y}{b} + \frac{x}{a} = 1$$

Rearranging:

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

Step 5: Alternative conceptual interpretation.

Observe that the determinant structure behaves like a shifted diagonal perturbation matrix. The extra determinant contribution beyond abc comes from the linear variable terms:

$$abz, \quad acy, \quad bcx$$

Balancing this against the given determinant value immediately produces the normalized symmetric relation:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Quick Tip: Whenever determinants contain repeated rows or columns with slight shifts, use row subtraction immediately to reveal hidden factorization patterns.

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10. If

$$y = \left(x^{\sin x}\right)^{\tan x},$$

then find

$$\frac{dy}{dx}.$$

(A)

$$y\left[\sec^2 x\ \sin x\ \ln x + \tan x\ \cos x\ \ln x + \frac{\tan x\ \sin x}{x}\right]$$

(B)

$$y\,(\cos x + \sin x)$$

(C)

$$y \tan x$$

(D)

$$x^{\sin x}$$

Correct Answer: (A)

$$y\left[\sec^2 x \sin x \ln x + \tan x \cos x \ln x + \frac{\tan x \sin x}{x}\right]$$

Solution:

Concept: This is a highly composite logarithmic differentiation problem where the variable appears:

- in the base,
- inside trigonometric functions,

- and also in the exponent.

Such expressions are simplified only through repeated logarithmic transformation and systematic implicit differentiation.

Step 1: Simplifying the given function.

Given:

$$y = \left(x^{\sin x}\right)^{\tan x}$$

Using:

$$(a^m)^n = a^{mn}$$

we get:

$$y = x^{\sin x \tan x}$$

Now observe:

$$\sin x \tan x = \sin x \left(\frac{\sin x}{\cos x}\right) = \frac{\sin^2 x}{\cos x}$$

However, keeping the exponent in product form is more convenient for differentiation.

Thus,

$$y = x^{\sin x \tan x}$$

Step 2: Taking logarithm on both sides.

Applying natural logarithm:

$$\ln y = \sin x \tan x \ln x$$

Now the expression becomes a product of three functions:

$$(\sin x)(\tan x)(\ln x)$$

Step 3: Differentiating implicitly.

Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}[\sin x \tan x \ln x]$$

Apply product rule carefully for three factors.

Step 4: Differentiating each factor.

Using:

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Hence,

$$\frac{1}{y}\frac{dy}{dx} = (\cos x)(\tan x)(\ln x) + (\sin x)(\sec^2 x)(\ln x) + (\sin x)(\tan x)\left(\frac{1}{x}\right)$$

Rearranging terms:

$$\frac{1}{y}\frac{dy}{dx} = \sec^2 x \sin x \ln x + \tan x \cos x \ln x + \frac{\tan x \sin x}{x}$$

Step 5: Obtaining the final derivative.

Multiply both sides by y:

$$\frac{dy}{dx} = y\left[\sec^2 x \sin x \ln x + \tan x \cos x \ln x + \frac{\tan x \sin x}{x}\right]$$

Therefore,

$$\boxed{\frac{dy}{dx} = y\left[\sec^2 x \sin x \ln x + \tan x \cos x \ln x + \frac{\tan x \sin x}{x}\right]}$$

Step 6: Why ordinary differentiation becomes difficult.

If logarithmic differentiation were not used, direct differentiation would involve repeated exponential chain rules and implicit structures simultaneously. Logarithmic differentiation converts complicated exponential expressions into manageable algebraic differentiation.

Quick Tip: For nested exponential expressions involving trigonometric exponents, logarithmic differentiation is almost always the fastest and safest method.
