# CUET 2026 May 22 Shift 2 Mathematics

## Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

### General Instructions

**(i)** The examination will be conducted in Computer-Based Test (CBT) mode.

**(ii)** Each question carries +5 marks for a correct answer and -1 mark for a wrong answer.

**(iii)** The total number of questions is 50.

**(iv)** Duration of the examination is 1 hour (60 minutes).

**1. If $A$ is a non-singular square matrix of order $3 \times 3$ such that its determinant is $|A| = 5$, find the absolute value of the determinant of its adjoint matrix, represented as $|\text{adj}(A)|$.**

(A) 5

(B) 125

(C) 25

(D) 15

**Correct Answer:** (C) 25

**Solution:**

**Concept:** For any non-singular square matrix $A$ of order $n \times n$, the determinant of its adjoint matrix is directly related to the determinant of the original matrix through the fundamental algebraic identity:

$$|\text{adj}(A)| = |A|^{n-1}$$

This property allows us to calculate the value without needing to compute the individual elements of the adjoint matrix.

**Step 1:** Identify the matrix order and determinant from the problem values.

The problem provides the following parameters:

- Matrix order $(n) = 3$

- Determinant value ($|A|$) = 5

**Step 2:** Substitute parameters into the adjoint determinant identity.

Plug these values directly into the scaling formula:

$$|\text{adj}(A)| = 5^{3-1} = 5^2$$

**Step 3:** Simplify the exponential expression.

Evaluating the exponent yields the final scalar determinant value:

$$|\text{adj}(A)| = 25$$

**Quick Tip:** Keep this related identity handy for multi-step matrix problems: the determinant of the adjoint of an adjoint matrix scales even higher, following the rule **$|\text{adj}(\text{adj}(A))| = |A|^{(n-1)^2}$**.

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**2. Determine the exact expression for the Integrating Factor (I.F.) of the following first-order linear differential equation: $\frac{dy}{dx} - y\tan x = e^x$**

(A) $\sec x$

(B) $\cos x$

(C) $\sin x$

(D) $e^{-\tan x}$

**Correct Answer:** (B) $\cos x$

**Solution:**

**Concept:** A first-order linear differential equation written in standard form is expressed as:

$$\frac{dy}{dx} + Py = Q$$

Where $P$ and $Q$ are functions of $x$ or constants. The **Integrating Factor (I.F.)** needed to solve this class of differential equations is given by the calculus formula:

$$\text{I.F.} = e^{\int P\,dx}$$

**Step 1:** Isolate the coefficient function $P$ from the equation structure.

Comparing our given equation to the standard linear format shows:

$$P = -\tan x$$

*(Note: It is crucial to include the negative sign attached to the tangent function).*

**Step 2:** Evaluate the indefinite integral of $P$.

Set up and integrate the tangent function with respect to $x$:

$$\int P\,dx = \int -\tan x\,dx = -\ln|\sec x|$$

Using logarithmic properties, bring the negative sign inside as a reciprocal power:

$$-\ln|\sec x| = \ln\left|\frac{1}{\sec x}\right| = \ln|\cos x|$$

**Step 3:** Substitute the integrated term back into the exponential base.

Plug the evaluated integral into the final integrating factor template:

$$\text{I.F.} = e^{\ln|\cos x|}$$

Since the exponential base $e$ and natural logarithm ln cancel each other out, the expression simplifies to:

$$\text{I.F.} = \cos x$$

**Quick Tip:** Always ensure your linear differential equation is in standard form before identifying $P$. The coefficient of the leading $\frac{dy}{dx}$ term must equal exactly 1. If it doesn't, divide the entire equation by that term first.

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**3. Find the open interval across which the cubic polynomial function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is classified as strictly decreasing.**

(A) $(-2, 3)$

(B) $(-\infty, -2) \cup (3, \infty)$

(C) $(-3, 2)$

(D) $(0, \infty)$

**Correct Answer:** (A) (-2, 3)

**Solution:**

**Concept:** A continuous function $f(x)$ is strictly decreasing across an open interval if its first derivative is strictly negative ($f'(x) < 0$) at every point within that interval.

**Step 1:** Calculate the first derivative of the polynomial function.

Apply the standard power derivative rule to each term of the polynomial:

$$f'(x) = \frac{d}{dx}\left(2x^3 - 3x^2 - 36x + 7\right) = 6x^2 - 6x - 36$$

**Step 2:** Find the critical roots by factoring the quadratic derivative expression.

Set the derivative expression equal to zero to isolate the boundary roots:

$$6x^2 - 6x - 36 = 0 \quad \Rightarrow \quad 6\left(x^2 - x - 6\right) = 0$$

Factor the quadratic trinomial inside the parentheses:

$$6(x-3)(x+2) = 0$$

This isolates the critical turning points at $x = 3$ and $x = -2$.

**Step 3:** Analyze the derivative signs using the wavy curve method.

The critical points divide the real number line into three separate intervals: $(-\infty, -2)$, $(-2, 3)$, and $(3, \infty)$.

- For $x \in (-\infty, -2)$, $f'(x) > 0$ (Strictly Increasing)
- For $x \in (-2, 3)$, $f'(x) < 0$ (Strictly Decreasing)
- For $x \in (3, \infty)$, $f'(x) > 0$ (Strictly Increasing)

Thus, the function is strictly decreasing on the open interval $(-2, 3)$.

**Quick Tip:** When evaluating quadratic inequalities of the form $(x-a)(x-b) < 0$ where $a < b$, the solution space will always be the interior bounded range **$(a, b)$**.

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**4. Find the shortest distance between the two parallel straight lines whose vector position**

equations are given by:

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

(A) $\sqrt{2}$

(B) $\frac{\sqrt{293}}{7}$

(C) 0

(D) $\frac{5}{7}$

**Correct Answer:** (B) $\frac{\sqrt{293}}{7}$

**Solution:**

**Concept:** When two straight lines run completely parallel in a 3D space, they share the exact same directional heading vector $\vec{b}$. The shortest distance $(d)$ separating them depends on the cross product of their position difference with this direction vector:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

**Step 1:** Extract the vector parameters and calculate the position difference.

From the provided parallel equations, isolate the coordinates:

- $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$
- $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$
- Shared Direction Vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Calculate the initial displacement vector connecting the two starting points:

$$\vec{a}_2 - \vec{a}_1 = (3-1)\hat{i} + (3-2)\hat{j} + (-5-(-4))\hat{k} = 2\hat{i} + \hat{j} - \hat{k}$$

**Step 2:** Compute the cross product vector $(\vec{a}_2 - \vec{a}_1) \times \vec{b}$.

Set up the matrix determinant layout using standard unit coordinates:

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

Expanding along the top row:

$$= \hat{i}(6-(-3)) - \hat{j}(12-(-2)) + \hat{k}(6-2) = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

Calculate its magnitude:

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{9^2 + (-14)^2 + 4^2} = \sqrt{81 + 196 + 16} = \sqrt{293}$$

**Step 3:** Divide by the magnitude of the direction vector $|\vec{b}|$.

Find the magnitude of the shared direction heading:

$$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Combine these values into the parallel distance formula:

$$d = \frac{\sqrt{293}}{7}$$

**Quick Tip:** Always double-check the direction vectors before starting. If they match or are direct scalar multiples of each other, the lines are parallel. You must use the **parallel cross product formula**, not the standard skew-lines formula.

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**5. Find the maximum value of the linear objective optimization function**

$$Z = 4x + y$$

**evaluated over a feasible region bounded by the corner vertices:**

$$(0,0),\ (3,0),\ (2,3),\ \textbf{and } (0,4).$$

(A) 12

(B) 4

(C) 11

(D) 16

**Correct Answer:** (A) 12

**Solution:**

**Concept:** In Linear Programming Problems (LPP), the maximum or minimum value of an objective function occurs at one of the corner points (vertices) of the feasible region.

**Step 1:** Identify the objective function.

The objective function is:

$$Z = 4x + y$$

The feasible region has the corner points:

$$(0,0),\ (3,0),\ (2,3),\ (0,4)$$

**Step 2:** Evaluate the objective function at each corner point.

$$Z(0,0) = 4(0) + 0 = 0$$

$$Z(3,0) = 4(3) + 0 = 12$$

$$Z(2,3) = 4(2) + 3 = 8 + 3 = 11$$

$$Z(0,4) = 4(0) + 4 = 4$$

**Step 3:** Determine the maximum value.

The obtained values are:

$$0,\ 12,\ 11,\ 4$$

The maximum among these is:

$$12$$

Hence, the maximum value of the objective function is:

$$\boxed{12}$$

Therefore, the correct answer is:

$$\boxed{\text{(A)}}$$

**Quick Tip:** For Linear Programming Problems, always evaluate the objective function at all corner points of the feasible region. The optimal value always occurs at a vertex.

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**6. If $A$ is a square matrix of order 3 such that its determinant is $|A| = 3$, calculate the value of the scalar matrix determinant represented by $|2A|$.**

(A) 6

(B) 24

(C) 12

(D) 18

**Correct Answer:** (B) 24

**Solution:**

**Concept:** For any square matrix $A$ of order $n \times n$, factoring out a scalar multiplier $k$ from inside a determinant requires raising that scalar to the power of the matrix order:

$$|kA| = k^n |A|$$

This occurs because the scalar multiplier scales every individual row of the matrix uniformly.

**Step 1:** Identify the scale factor, matrix order, and base determinant.

The problem provides the following parameters:

- Scalar factor $(k) = 2$
- Matrix order $(n) = 3$
- Base determinant $(|A|) = 3$

**Step 2:** Apply parameters to the determinant scaling property.

Substitute these values into the scaling identity:

$$|2A| = 2^3 \times |A|$$

**Step 3:** Simplify the numerical expression.

Evaluate the cubic exponent and multiply by the base determinant:

$$|2A| = 8 \times 3 = 24$$

**Quick Tip:** Remember to always check the order of the matrix ($n$) before scaling a determinant. Forgetting to raise the scalar factor to the power of $n$ is a very common exam mistake.

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**7. An unbiased coin is tossed twice. Let event $A$ represent getting a head on the first toss, and event $B$ represent getting a head on the second toss. Determine the mathematical relationship between events $A$ and $B$.**

(A) Dependent Events

(B) Independent Events

(C) Mutually Exclusive Events

(D) Equivalence Events

**Correct Answer:** (B) Independent Events

**Solution:**

**Concept:** Two events $A$ and $B$ are mathematically classified as **Independent Events** if and only if the occurrence of one does not affect the probability of the other. This condition can be verified using the multiplication rule:

$$P(A \cap B) = P(A) \cdot P(B)$$

**Step 1:** Define the sample space and calculate individual event probabilities.

Tossing an unbiased coin twice produces a sample space containing 4 equally likely outcomes:

$$S = \{\text{HH, HT, TH, TT}\} \quad \Rightarrow \quad n(S) = 4$$

Map the outcomes for each event and calculate their probabilities:

- Event $A$ (Head on 1st toss) $= \{\text{HH}, \text{HT}\} \Rightarrow P(A) = \frac{2}{4} = 0.5$
- Event $B$ (Head on 2nd toss) $= \{\text{HH}, \text{TH}\} \Rightarrow P(B) = \frac{2}{4} = 0.5$

**Step 2:** Calculate the joint intersection probability of both events.

The intersection event $A \cap B$ represents getting a head on both the first and second tosses:

$$A \cap B = \{\text{HH}\} \quad \Rightarrow \quad P(A \cap B) = \frac{1}{4} = 0.25$$

**Step 3:** Verify the independent multiplication identity.

Multiply the individual event probabilities together:

$$P(A) \cdot P(B) = 0.5 \times 0.5 = 0.25$$

Since $P(A \cap B) = P(A) \cdot P(B) = 0.25$, the two events are independent.

**Quick Tip:** Do not confuse independent events with mutually exclusive events. Independent events can happen at the same time ($P(A \cap B) \neq 0$), whereas mutually exclusive events can never occur together ($P(A \cap B) = 0$).

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**8. Find the equation of the normal to the curve $y = x^2 - x$ at the coordinate point position $(1, 0)$.**

(A) $x + y - 1 = 0$

(B) $x - y - 1 = 0$

(C) $x + y + 1 = 0$

(D) $2x + y - 2 = 0$

**Correct Answer:** (A) x + y - 1 = 0

**Solution:**

**Concept:** The slope of the tangent line ($m_t$) to a curve at a given point is found by calculating its first derivative at that point. Because a normal line runs perpendicular to the tangent, its slope ($m_n$) is the negative reciprocal of the tangent slope:

$$m_n = -\frac{1}{m_t} = -\frac{1}{\left(\frac{dy}{dx}\right)}$$

**Step 1:** Calculate the first derivative and find the tangent slope.

Differentiate the curve's equation with respect to $x$:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - x\right) = 2x - 1$$

Evaluate this derivative at the target x-coordinate ($x = 1$) to find the tangent slope:

$$m_t = 2(1) - 1 = 1$$

**Step 2:** Calculate the perpendicular slope of the normal line.

Take the negative reciprocal of the tangent slope:

$$m_n = -\frac{1}{1} = -1$$

**Step 3:** Set up the line equation using point-slope form.

Using the point coordinates $(1, 0)$ and our normal slope $m_n = -1$, write the line equation:

$$y - y_1 = m_n(x - x_1) \quad \Rightarrow \quad y - 0 = -1(x - 1)$$

Distribute the negative sign and rearrange all terms to the left side:

$$y = -x + 1 \quad \Rightarrow \quad x + y - 1 = 0$$

**Quick Tip:** Always read the question carefully to see if it asks for the equation of the **tangent** or the **normal**. Forgetting to invert and negate the slope for a normal line is a common oversight under exam time pressure.

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**9. Evaluate the value of the following definite integral using standard calculus integrations:**

$\int_0^1 xe^x \, dx$

(A) $e$

(B) 1

(C) $e - 1$

(D) 0

**Correct Answer:** (B) 1

**Solution:**

**Concept:** When an integrand consists of a product of two distinct functions, apply the **Integration by Parts** rule:

$$\int u\,dv = uv - \int v\,du$$

Choose the parts systematically using the **ILATE** priority rule (Inverses, Logarithms, Algebraics, Trigonometrics, Exponentials).

**Step 1:** Assign integration variables following the ILATE rule.

For the integrand $xe^x$, choose the algebraic term as $u$ and the exponential term as $dv$:

- Let $u = x \quad \Rightarrow \quad du = dx$
- Let $dv = e^x\,dx \quad \Rightarrow \quad v = e^x$

**Step 2:** Apply the integration by parts formula.

Substitute these terms into the integration by parts formula:

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$

Evaluate the remaining integral:

$$\int xe^x\,dx = xe^x - e^x = e^x(x-1)$$

**Step 3:** Evaluate the definite integral boundaries.

Apply the limits from 0 to 1 across the integrated expression:

$$\int_0^1 xe^x\,dx = [e^x(x-1)]_0^1$$

Evaluate the upper limit $(x = 1)$ and subtract the lower limit $(x = 0)$:

$$= \left(e^1(1-1)\right) - \left(e^0(0-1)\right)$$

$$= 0 - (1\cdot(-1)) = 0 - (-1) = 1$$

**Quick Tip:** Be careful when evaluating lower limits at zero, especially with exponential functions. While algebraic variables like $x$ drop to 0, exponential terms evaluate to 1 ($e^0 = 1$), which often changes the final result.

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**10. Find the total number of distinct binary relations that can be defined over a set $A$ containing exactly 3 elements.**

(A) 9

(B) 64

(C) 512

(D) 27

**Correct Answer:** (C) 512

**Solution:**

**Concept:** A binary relation defined on a set $A$ is any subset of the Cartesian product set $A \times A$. Therefore, the total number of distinct relations is equal to the total number of subsets in the power set of $A \times A$, calculated using the formula:

$$\text{Total Relations} = 2^{n(A\times A)} = 2^{n^2}$$

Where $n$ represents the total number of elements in set $A$.

**Step 1:** Calculate the size of the Cartesian product set.

The problem states that set $A$ contains exactly 3 elements $(n = 3)$. Find the total number of coordinate pairs in the Cartesian product:

$$n(A \times A) = n \times n = 3 \times 3 = 9$$

**Step 2:** Calculate the total number of relation power subsets.

Raise 2 to the power of the Cartesian product size to find the total number of possible subsets:

$$\text{Total Relations} = 2^9$$

**Step 3:** Evaluate the final exponential value.

Multiplying out the base-2 value yields:

$$2^9 = 512$$

**Quick Tip:** Keep these related relation counting formulas memorized for matching questions:

- Total Relations $= 2^{n^2}$
- Total Reflexive Relations $= 2^{n^2-n}$
- Total Symmetric Relations $= 2^{\frac{n(n+1)}{2}}$

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