CUET 2026 May 29 Shift 1 Mathematics

Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. Let A be a 3×3 matrix such that $A^2 = I$. If $\det(A) = -1$ and the sum of eigenvalues is 1, find the set of eigenvalues of A.

(A) $1, 1, 1$

(B) $1, 1, -1$

(C) $-1, -1, 1$

(D) $-1, -1, -1$

Correct Answer: (B) $1, 1, -1$

Solution:

Concept: An involutory matrix satisfies $A^2 = I$. Hence, every eigenvalue λ of A satisfies:

$$\lambda^2 = 1$$

Therefore, the eigenvalues can only be 1 or -1.

Step 1: Use the determinant condition.

The determinant of a matrix equals the product of its eigenvalues:

$$\lambda_1\lambda_2\lambda_3 = -1$$

Thus, an odd number of eigenvalues must be negative.

Step 2: Use the trace condition.

The sum of eigenvalues is:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

Step 3: Check possible combinations.

Possible eigenvalue sets are:

$$(1,1,-1), \quad (-1,-1,-1)$$

For $(1,1,-1)$:

$$1+1-1=1$$

and

$$1 \cdot 1 \cdot (-1) = -1$$

Both conditions are satisfied.

Quick Tip: For matrices satisfying $A^2 = I$, eigenvalues are always restricted to ± 1. Use determinant and trace conditions together to uniquely identify them.

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2. Analyze the function $f(x) = |x-1| + |x-2| + |x-3|$. Determine the points where the derivative $f'(x)$ is undefined.

(A) $x = 1, 3$

(B) $x = 1, 2, 3$

(C) $x = 1.5, 2.5$

(D) $x = 0$

Correct Answer: (B) $x = 1, 2, 3$

Solution:

Concept: The function $|x-a|$ is non-differentiable at $x = a$ because the left-hand derivative and right-hand derivative are different at that point.

Step 1: Identify each absolute value term.

The given function is:

$$f(x) = |x-1| + |x-2| + |x-3|$$

Step 2: Locate possible non-differentiable points.

Each term becomes non-differentiable where its argument becomes zero:

$$x-1=0 \Rightarrow x=1$$

$$x-2=0 \Rightarrow x=2$$

$$x-3=0 \Rightarrow x=3$$

Step 3: Conclude the result.

Thus, the derivative $f'(x)$ is undefined at:

$$x=1,2,3$$

Quick Tip: Absolute value functions are differentiable everywhere except at the point where the expression inside the modulus becomes zero.

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3. Identify the order and degree of the differential equation:

$$\left(\frac{d^3y}{dx^3}\right)^2+4\left(\frac{dy}{dx}\right)^4+y=\sin(x)$$

(A) Order $3,$ Degree 4

(B) Order $3,$ Degree 2

(C) Order $4,$ Degree 3

(D) Order $1,$ Degree 4

Correct Answer: (B) Order $3,$ Degree 2

Solution:

Concept: The order of a differential equation is the order of the highest derivative present. The degree is the exponent of the highest order derivative after removing radicals and fractions involving derivatives.

Step 1: Find the highest order derivative.

The highest derivative present is:

$$\frac{d^3y}{dx^3}$$

Hence, the order is:

$$3$$

Step 2: Determine the degree.

The highest order derivative appears as:

$$\left(\frac{d^3y}{dx^3}\right)^2$$

Therefore, the degree is:

$$2$$

Quick Tip: Degree is always determined using the highest order derivative only, not by the largest exponent appearing elsewhere in the equation.

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4. For the linear differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = x^2$$

calculate the integrating factor (I.F.).

(A) x

(B) x^2

(C) x^3

(D) $\ln(x)$

Correct Answer: (B) x^2

Solution:

Concept: For a linear differential equation:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

the integrating factor is:

$$I.F. = e^{\int P(x)\,dx}$$

Step 1: Identify $P(x)$.

Comparing with the standard form:

$$P(x) = \frac{2}{x}$$

Step 2: Calculate the integral.

$$\int \frac{2}{x} dx = 2\ln|x|$$

Step 3: Find the integrating factor.

$$I.F. = e^{2\ln|x|}$$

Using logarithmic properties:

$$e^{2\ln|x|} = e^{\ln(x^2)} = x^2$$

Hence, the integrating factor is:

$$x^2$$

Quick Tip: For equations of the form $\frac{dy}{dx} + \frac{n}{x}y = Q(x)$, the integrating factor is usually x^n.

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5. Maximize $Z = 5x + 3y$ subject to $x + y \leq 6$ and $x, y \geq 0$. Where does the maximum value occur?

(A) $(0,0)$

(B) $(0,6)$

(C) $(6,0)$

(D) $(3,3)$

Correct Answer: (C) $(6,0)$

Solution:

Concept: In Linear Programming Problems, the optimum value occurs at the corner points of the feasible region.

Step 1: Find the feasible region vertices.

The constraints are:

$$x + y \leq 6, \quad x \geq 0, \quad y \geq 0$$

Thus, the corner points are:

$$(0,0),\ (6,0),\ (0,6)$$

Step 2: Evaluate the objective function.

At $(0,0)$:

$$Z = 5(0) + 3(0) = 0$$

At $(6,0)$:

$$Z = 5(6) + 3(0) = 30$$

At $(0,6)$:

$$Z = 5(0) + 3(6) = 18$$

Step 3: Determine the maximum value.

The maximum value is:

$$30$$

which occurs at:

$$(6,0)$$

Quick Tip: In two-variable LPP problems, always evaluate the objective function only at the corner points of the feasible region.

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6. A bag has 4 red and 6 black balls. Two balls are drawn without replacement. What is the probability that the second is red given the first was black?

(A) 4/10

(B) 4/9

(C) 6/9

(D) 2/9

Correct Answer: (B) 4/9

Solution:

Concept: Conditional probability is used when one event has already occurred. If sampling is done without replacement, the total number of objects decreases after each draw.

Step 1: Identify the initial composition of the bag:

Initially,

$$\text{Red balls} = 4, \qquad \text{Black balls} = 6$$

Total balls:

$$4 + 6 = 10$$

Step 2: Apply the given condition:

The first ball drawn is black. Hence one black ball is removed.

Remaining balls:

$$\text{Red} = 4, \qquad \text{Black} = 5$$

Total remaining:

$$4 + 5 = 9$$

Step 3: Calculate the required probability:

Probability that the second ball is red:

$$P(\text{Second is Red} \mid \text{First is Black}) = \frac{4}{9}$$

Quick Tip: In conditional probability problems with "without replacement," always update both the numerator and denominator after the first event occurs.

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7. Find the domain of the function $f(x) = \sin^{-1}(3x - 1)$.

(A) $[-1, 1]$

(B) $[0, \frac{2}{3}]$

(C) $[-\frac{1}{3}, \frac{1}{3}]$

(D) $[\frac{1}{3}, 1]$

Correct Answer: (B) $[0, \frac{2}{3}]$

Solution:

Concept: The inverse sine function is defined only when its argument lies in the interval:

$$-1 \le u \le 1$$

Step 1: Apply the inverse sine condition:

For

$$f(x) = \sin^{-1}(3x - 1)$$

we require:

$$-1 \le 3x - 1 \le 1$$

Step 2: Solve the inequality:

Add 1 throughout:

$$0 \le 3x \le 2$$

Divide by 3:

$$0 \le x \le \frac{2}{3}$$

Step 3: Write the domain:

Hence the domain is:

$$\boxed{\left[0, \frac{2}{3}\right]}$$

Quick Tip: For inverse trigonometric functions, always restrict the inner expression according to the valid range of the original trigonometric function.

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8. For vectors $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, find the scalar projection of $\vec{a}$ onto $\vec{b}$.

(A) $-\frac{1}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{6}}$

(C) $-\frac{1}{6}$

(D) $\frac{1}{6}$

Correct Answer: (A) $-\frac{1}{\sqrt{6}}$

Solution:

Concept: The scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is given by:

$$\text{Scalar Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

Step 1: Find the dot product:

$$\vec{a}\cdot\vec{b} = (3)(1) + (-1)(2) + (2)(-1)$$

$$= 3 - 2 - 2 = -1$$

Step 2: Find the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + (-1)^2}$$

$$= \sqrt{1+4+1} = \sqrt{6}$$

Step 3: Calculate scalar projection:

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{-1}{\sqrt{6}}$$

Quick Tip: A negative scalar projection means the vector has a component opposite to the direction of the reference vector.

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9. Let R be a relation on $\{1,2,3\}$ defined by

$$R = \{(1,1),(2,2),(3,3),(1,2)\}$$

Identify the properties satisfied by R.

(A) Symmetric only

(B) Reflexive and Transitive

(C) Equivalence relation

(D) None of these

Correct Answer: (B) Reflexive and Transitive

Solution:

Concept: A relation is:

- Reflexive if every element is related to itself.
- Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$.
- Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.

Step 1: Check reflexivity:

Since

$$(1,1),(2,2),(3,3) \in R$$

the relation is reflexive.

Step 2: Check symmetry:

We have:

$$(1,2) \in R$$

but

$$(2,1) \notin R$$

Hence the relation is not symmetric.

Step 3: Check transitivity:

The only nontrivial ordered pair is $(1,2)$. Since there is no pair beginning with 2 except $(2,2)$, transitivity is satisfied:

$$(1,2),(2,2) \Rightarrow (1,2) \in R$$

Thus the relation is transitive.

Quick Tip: If a relation contains all identity pairs and very few cross-pairs, it is often reflexive and transitive but fails symmetry.

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10. Find the local maximum point of the function

$$f(x) = -x^3 + 3x + 1$$

(A) $x = 1$

(B) $x = -1$

(C) $x = 0$

(D) $x = \sqrt{3}$

Correct Answer: (A) $x = 1$

Solution:

Concept: Critical points occur where:

$$f'(x) = 0$$

The second derivative test determines whether the point is a maximum or minimum:

$$f''(x) < 0 \Rightarrow \text{Local Maximum}$$

$$f''(x) > 0 \Rightarrow \text{Local Minimum}$$

Step 1: Differentiate the function:

$$f(x) = -x^3 + 3x + 1$$

$$f'(x) = -3x^2 + 3$$

Step 2: Find the critical points:

Set:

$$-3x^2 + 3 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Step 3: Apply second derivative test:

$$f''(x) = -6x$$

At $x = 1$:

$$f''(1) = -6 < 0$$

Hence $x = 1$ is a local maximum point.

At $x = -1$:

$$f''(-1) = 6 > 0$$

Hence $x = -1$ is a local minimum point.

Quick Tip: A negative second derivative indicates downward concavity, which corresponds to a local maximum.
