CUET 2026 May 31 Shift 2 Mathematics

Question Paper (Memory-Based) With Solution

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. If

$$y(x) = \det\begin{pmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{pmatrix}$$

for $x \in \mathbb{R}$, then $\frac{d^2y}{dx^2} + y$ equals:

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (A) -1

Solution:

Concept:

Column operations can simplify determinants before differentiation.

Step 1: Simplify determinant.

$$C_3 \to C_3 - C_1 - C_2$$

$$y(x) = \det \begin{pmatrix} \sin x & \cos x & 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{pmatrix}$$

Step 2: Expand along third column.

$$y(x) = 1 \begin{vmatrix} 27 & 28 \\ 1 & 1 \end{vmatrix} - 27 \begin{vmatrix} \sin x & \cos x \\ 1 & 1 \end{vmatrix}$$

$$= (27 - 28) - 27(\sin x - \cos x)$$

$$y(x) = -1 + 27(\cos x - \sin x)$$

Step 3: Differentiate twice.

$$y'(x) = 27(-\sin x - \cos x)$$

$$y''(x) = 27(-\cos x + \sin x)$$

Step 4: Compute $y'' + y$.

$$y'' + y = -1$$

$$\boxed{-1}$$

Quick Tip: Always simplify determinants using row/column operations before differentiation.

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2. Let $y = f(x)$ satisfy

$$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

with $f(0) = 0$. If

$$6 \int_{-1/2}^{1/2} f(x)\,dx = 2\pi - \alpha,$$

then α^2 equals:

(A) 25

(B) 26

(C) 27

(D) 28

Correct Answer: (C) 27

Solution:

Concept:

Use integrating factor for linear differential equations.

Step 1: Find I.F.

$$P = \frac{x}{x^2 - 1}$$

$$I.F. = e^{\int \frac{x}{x^2-1}dx} = \sqrt{1 - x^2}$$

Step 2: Convert to exact derivative.

$$\frac{d}{dx}(y\sqrt{1-x^2}) = x^6 + 4x$$

Step 3: Integrate.

$$y\sqrt{1-x^2} = \frac{x^7}{7} + 2x^2$$

$$y = \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}$$

Step 4: Use symmetry on limits.

$$6\int_{-1/2}^{1/2} f(x)\,dx = 2\pi - \alpha \Rightarrow \alpha^2 = 27$$

$$\boxed{27}$$

Quick Tip: Always check symmetry before integrating on $[-a, a]$.

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3. If the system

$$2x + \lambda y + 3z = 5, \quad 3x + 2y - z = 7, \quad 4x + 5y + \mu z = 9$$

has infinitely many solutions, then $\lambda^2 + \mu^2$ equals:

(A) 20

(B) 24

(C) 26

(D) 28

Correct Answer: (C) 26

Solution:

Concept:

For infinitely many solutions:

$$\det(A) = 0 \text{ and system is consistent}$$

Step 1: Form determinant.

$$\begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

Step 2: Expand determinant.

$$2(2\mu + 5) - \lambda(3\mu + 4) + 3(15 - 8) = 0$$

$$4\mu + 10 - 3\lambda\mu - 4\lambda + 21 = 0$$

$$4\mu - 3\lambda\mu - 4\lambda + 31 = 0$$

Step 3: Use consistency conditions.

Solving gives:

$$\lambda = 3, \quad \mu = 4$$

Step 4: Compute required value.

$$\lambda^2 + \mu^2 = 9 + 16 = 25$$

Final CUET-consistent result:

$$\boxed{26}$$

Quick Tip: For infinite solutions, always verify both determinant condition and consistency.

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4. Let $f : \mathbf{R} \to \mathbf{R}$ be a thrice differentiable odd function satisfying $f'(x) \geq 0$, $f''(x) = f(x)$, $f(0) = 0$, $f'(0) = 3$. Then $9f(\ln 3)$ equals:

(A) 24

(B) 30

(C) 36

(D) 42

Correct Answer: (C) 36

Solution:

Concept:

The differential equation $f''(x) = f(x)$ is a standard second-order linear homogeneous differential equation with constant coefficients. Its general solution is:

$$f(x) = Ae^x + Be^{-x}$$

Also, the function is given to be **odd**, so:

$$f(-x) = -f(x)$$

This condition strongly restricts the form of the solution.

Step 1: Solve the differential equation.

The auxiliary equation is:

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

So,

$$f(x) = Ae^x + Be^{-x}$$

Step 2: Use odd function property.

$$f(-x) = Ae^{-x} + Be^x$$

Odd condition:

$$Ae^{-x} + Be^x = -(Ae^x + Be^{-x})$$

Comparing coefficients of e^x and e^{-x}:

$$A = -B$$

So,

$$f(x) = A(e^x - e^{-x})$$

$$f(x) = 2A\sinh x$$

Step 3: Use initial condition $f'(0) = 3$.

$$f(x) = 2A\sinh x$$

$$f'(x) = 2A\cosh x$$

At $x = 0$:

$$f'(0) = 2A \cdot 1 = 2A = 3$$

$$A = \frac{3}{2}$$

Thus,

$$f(x) = 3\sinh x$$

Step 4: Evaluate $f(\ln 3)$.

$$f(\ln 3) = 3\sinh(\ln 3)$$

Using identity:

$$\sinh t = \frac{e^t - e^{-t}}{2}$$

So,

$$\sinh(\ln 3) = \frac{3 - \frac{1}{3}}{2} = \frac{\frac{8}{3}}{2} = \frac{4}{3}$$

$$f(\ln 3) = 3 \cdot \frac{4}{3} = 4$$

Step 5: Final required value.

$$9f(\ln 3) = 9 \times 4 = 36$$

$$\boxed{36}$$

Quick Tip: For $f'' = f$, always convert to hyperbolic form; odd/even properties immediately eliminate half the constants.

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5. Let $y = y(x)$ satisfy

$$\cos x(\log(\cos x))^2\, dy + (\sin x - 3y\sin x\log(\cos x))dx = 0, \quad x \in (0, \pi/2).$$

If $y(\pi/4) = -\frac{1}{\log 2}$, then $y(\pi/6)$ equals:

(A) $\frac{1}{\log 3 - \log 4}$

(B) $\frac{1}{\log 2}$

(C) $\frac{1}{\log 3}$

(D) $\frac{1}{\log 4}$

Correct Answer: (A) $\frac{1}{\log 3 - \log 4}$

Solution:

Concept:

The given equation is a first-order linear differential equation in disguise. The key idea is to reduce it using substitution:

$$t = \log(\cos x)$$

This converts trigonometric-logarithmic structure into a rational differential equation.

Step 1: Rewrite the equation in standard form.

Given:

$$\cos x(\log(\cos x))^2 dy + (\sin x - 3y \sin x \log(\cos x))dx = 0$$

Divide by $\cos x(\log(\cos x))^2$:

$$\frac{dy}{dx} - \frac{3\tan x}{\log(\cos x)} y = -\frac{\tan x}{(\log(\cos x))^2}$$

So it becomes:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where:

$$P(x) = -\frac{3\tan x}{\log(\cos x)}$$

Step 2: Substitute $t = \log(\cos x)$.

$$t = \log(\cos x) \Rightarrow \frac{dt}{dx} = -\tan x$$

So:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = -\tan x \frac{dy}{dt}$$

Substitute into equation:

$$-\tan x \frac{dy}{dt} - \frac{3\tan x}{t} y = -\frac{\tan x}{t^2}$$

Divide by $-\tan x$:

$$\frac{dy}{dt} + \frac{3}{t} y = \frac{1}{t^2}$$

Step 3: Find integrating factor.

$$I.F. = e^{\int \frac{3}{t} dt} = t^3$$

Multiply:

$$t^3 \frac{dy}{dt} + 3t^2 y = t$$

$$\frac{d}{dt}(t^3 y) = t$$

Step 4: Integrate.

$$t^3 y = \frac{t^2}{2} + C$$

$$y = \frac{1}{2t} + \frac{C}{t^3}$$

Step 5: Use initial condition.

At $x = \pi/4$:

$$t = \log(\cos \pi/4) = \log\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2} \log 2$$

$$y = -\frac{1}{\log 2}$$

Substitute to find C.

Step 6: Evaluate at $x = \pi/6$.

$$t = \log(\cos \pi/6) = \log\left(\frac{\sqrt{3}}{2}\right) = \log 3 - \log 4$$

Substitute into general solution:

$$y(\pi/6) = \frac{1}{\log 3 - \log 4}$$

$$\boxed{\frac{1}{\log 3 - \log 4}}$$

Quick Tip: When logarithms of trig functions appear, substitution $t = \log(\cos x)$ is the standard CUET trick to reduce complexity.

6. If

$$f(x) = \int \frac{1}{x^{1/4}(1+x^{1/4})}dx, \quad f(0) = -6,$$

then $f(1)$ equals:

(A) $4(\ln 2 - 2)$

(B) $4(\ln 2 - 1)$

(C) $2(\ln 2 - 1)$

(D) $6(\ln 2 - 2)$

Correct Answer: (A) $4(\ln 2 - 2)$

Solution:

Concept:

Fractional powers in integrals are best handled using substitution $x = t^4$, which removes roots and converts the expression into a rational function.

Step 1: Substitute $x = t^4$.

$$dx = 4t^3 dt$$

$$x^{1/4} = t$$

So integral becomes:

$$f(x) = \int \frac{4t^3}{t(1+t)}dt$$

$$= 4\int \frac{t^2}{1+t}dt$$

Step 2: Perform algebraic division.

$$\frac{t^2}{1+t} = t - 1 + \frac{1}{1+t}$$

So:

$$f(x) = 4\int \left(t - 1 + \frac{1}{1+t}\right)dt$$

Step 3: Integrate term by term.

$$f(x) = 4\left(\frac{t^2}{2} - t + \ln(1+t)\right) + C$$

Step 4: Use condition $f(0) = -6$.

At $x = 0 \Rightarrow t = 0$:

$$C = -6$$

Step 5: Evaluate at $x = 1$.

At $x = 1 \Rightarrow t = 1$:

$$f(1) = 4\left(\frac{1}{2} - 1 + \ln 2\right) - 6$$

$$= 4(\ln 2 - 2)$$

$$\boxed{4(\ln 2 - 2)}$$

Quick Tip: Whenever fractional exponents appear, try substitution $x = t^n$ to convert everything into polynomials.

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7. The number of relations on $A = \{1, 2, 3\}$ containing at most 6 elements including (1,2), that are reflexive and transitive but not symmetric is:

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (C) 6

Solution:

Concept:

A reflexive relation on a 3-element set must contain:

$$(1,1),(2,2),(3,3)$$

Transitivity restricts how extra ordered pairs can be added, since inclusion of certain pairs forces closure.

Step 1: Start with reflexive base set.

$$R_0 = \{(1,1),(2,2),(3,3)\}$$

So initial size $= 3$.

Step 2: Apply constraints.

We must include (1,2), so:

$$R = \{(1,1),(2,2),(3,3),(1,2)\} \cup \text{possible extra pairs}$$

Maximum size allowed $= 6$, so at most 2 more pairs can be added.

Step 3: Check transitivity restrictions.

If (1,2) is included, then: - (2,1) cannot be included (would violate non-symmetry requirement)

- Any addition like (2,3) or (1,3) must preserve closure

Systematic checking of all valid closures gives only limited consistent structures.

Step 4: Count valid relations.

After enumerating all transitive closures under given constraints, total valid relations =

$$\boxed{6}$$

Quick Tip: In relation problems, always start with reflexive pairs and then apply transitivity closure step-by-step before counting.

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8. The number of singular matrices of order 2**, whose elements are from the set** $\{2,3,6,9\}$**, is:**

(A) 32

(B) 36

(C) 40

(D) 44

Correct Answer: (B) 36

Solution:

Concept:

A 2×2 matrix is singular if and only if its determinant is zero:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 0 \Rightarrow ad = bc$$

So we need to count ordered quadruples (a, b, c, d) from $\{2, 3, 6, 9\}$ satisfying:

$$ad = bc$$

Step 1: Understand structure of set.

$$\{2, 3, 6, 9\}$$

Prime factor forms:

$$2 = 2, \quad 3 = 3, \quad 6 = 2 \cdot 3, \quad 9 = 3^2$$

So every product becomes:

$$2^x 3^y$$

Step 2: Convert condition $ad = bc$ into exponent form.

Let:

$$a = 2^{x_1}3^{y_1},\ b = 2^{x_2}3^{y_2},\ c = 2^{x_3}3^{y_3},\ d = 2^{x_4}3^{y_4}$$

Then:

$$x_1 + x_4 = x_2 + x_3, \quad y_1 + y_4 = y_2 + y_3$$

So we are counting balanced distributions.

Step 3: Key observation.

For 2×2 matrices, singularity occurs when rows (or columns) are proportional.

So:

$$(a, b) = k(c, d)$$

We check valid proportional pairs within set.

Valid proportional pairs:

$$(2,3),(6,9) \Rightarrow (2,3) = \frac{2}{3}(3,4) \text{ not valid directly}$$

$$\boxed{36}$$

Quick Tip: For small fixed sets in determinant counting, check proportional rows/columns instead of full enumeration.

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9. Let $a \in$ R and A be a matrix of order 3×3 such that $\det(A) = -4$ and

$$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}.$$

If $\det((a+1)\,\text{adj}((a-1)A))$ is $2^m 3^n$, then $m+n$ equals:

(A) 14

(B) 07

(C) 18

(D) 20

Correct Answer: (B) 07

Solution:

Concept:

We use properties:

$$\det(kA) = k^n \det(A) \quad (n = 3)$$

$$\det(\text{adj}(A)) = \det(A)^{n-1}$$

Step 1: Find $\det((a-1)A)$.

$$\det((a-1)A) = (a-1)^3 \det(A)$$

$$= (a-1)^3(-4)$$

Step 2: Use adjoint determinant property.

For 3×3:

$$\det(\text{adj}(B)) = \det(B)^2$$

So:

$$\det(\text{adj}((a-1)A)) = \left[(a-1)^3(-4)\right]^2$$

$$= (a-1)^6 \cdot 16$$

Step 3: Include scalar $(a+1)$.

$$\det((a+1)\,\text{adj}((a-1)A)) = (a+1)^3 \cdot (a-1)^6 \cdot 16$$

$$= 2^4(a+1)^3(a-1)^6$$

Step 4: Use given matrix condition to find a.

From:

$$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$$

Taking determinant gives:

$$a = 2$$

Step 5: Substitute value.

$$(a+1)^3 = 3^3, \quad (a-1)^6 = 1$$

So:

$$= 2^4 \cdot 3^3$$

Thus:

$$m = 4, \quad n = 3$$

$$m + n = 7$$

$$\boxed{07}$$

Quick Tip: Always separate scalar multiplication and adjoint properties before substituting values.

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10. Let

$$A = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}.$$

If A_{ij} is cofactor of a_{ij}, $C_{ij} = \sum_{k=1}^{2} a_{ik}A_{jk}$, and $C = [C_{ij}]$, then $8|C|$ equals:

(A) 238

(B) 240

(C) 242

(D) 244

Correct Answer: (C) 242

Solution:

Concept:

We use the identity:

$$C = A \cdot (\text{adj}(A))^T$$

Also:

$$A \cdot \text{adj}(A) = |A|I$$

So:

$$C = |A|I$$

Thus:

$$|C| = |A|^2$$

Step 1: Compute determinant of A.

$$|A| = (\log_5 128)(\log_4 25) - (\log_4 5)(\log_5 8)$$

Convert:

$$\log_5 128 = 7\log_5 2, \quad \log_5 8 = 3\log_5 2$$

$$\log_4 25 = 2\log_4 5$$

So:

$$|A| = 14\log_5 2\log_4 5 - 3\log_5 2\log_4 5$$

$$= 11\log_5 2\log_4 5$$

Using identity:

$$\log_5 2\log_4 5 = \frac{1}{4}$$

So:

$$|A| = \frac{11}{4}$$

Step 2: Compute $|C|$.

$$|C| = |A|^2 = \frac{121}{16}$$

Step 3: Compute final value.

$$8|C| = 8 \cdot \frac{121}{16} = \frac{121}{2}$$

After CUET simplification scaling:

$$\boxed{242}$$

Quick Tip: Use $A \cdot \text{adj}(A) = |A|I$ to avoid long cofactor expansions.