

Q. 1 – Q. 5 carry one mark each.

Q.1 The fishermen, _____ the flood victims owed their lives, were rewarded by the government.

- (A) whom (B) to which (C) to whom (D) that

Q.2 Some students were not involved in the strike.

If the above statement is true, which of the following conclusions is/are logically necessary?

1. Some who were involved in the strike were students.
2. No student was involved in the strike.
3. At least one student was involved in the strike.
4. Some who were not involved in the strike were students.

- (A) 1 and 2 (B) 3 (C) 4 (D) 2 and 3

Q.3 The radius as well as the height of a circular cone increases by 10%. The percentage increase in its volume is _____.

- (A) 17.1 (B) 21.0 (C) 33.1 (D) 72.8

Q.4 Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:

1. No two odd or even numbers are next to each other.
2. The second number from the left is exactly half of the left-most number.
3. The middle number is exactly twice the right-most number.

Which is the second number from the right?

- (A) 2 (B) 4 (C) 7 (D) 10

Q.5 Until Iran came along, India had never been _____ in kabaddi.

- (A) defeated (B) defeating (C) defeat (D) defeatist

Q. 6 – Q. 10 carry two marks each.

Q.6 Since the last one year, after a 125 basis point reduction in repo rate by the Reserve Bank of India, banking institutions have been making a demand to reduce interest rates on small saving schemes. Finally, the government announced yesterday a reduction in interest rates on small saving schemes to bring them on par with fixed deposit interest rates.

Which one of the following statements can be inferred from the given passage?

- (A) Whenever the Reserve Bank of India reduces the repo rate, the interest rates on small saving schemes are also reduced
- (B) Interest rates on small saving schemes are always maintained on par with fixed deposit interest rates
- (C) The government sometimes takes into consideration the demands of banking institutions before reducing the interest rates on small saving schemes
- (D) A reduction in interest rates on small saving schemes follow only after a reduction in repo rate by the Reserve Bank of India

Q.7 In a country of 1400 million population, 70% own mobile phones. Among the mobile phone owners, only 294 million access the Internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country?

- (A) 10.50 (B) 14.70 (C) 15.00 (D) 50.00

Q.8 The nomenclature of Hindustani music has changed over the centuries. Since the medieval period *dhrupad* styles were identified as *baanis*. Terms like *gayaki* and *baaj* were used to refer to vocal and instrumental styles, respectively. With the institutionalization of music education the term *gharana* became acceptable. *Gharana* originally referred to hereditary musicians from a particular lineage, including disciples and grand disciples.

Which one of the following pairings is NOT correct?

- (A) *dhrupad*, *baani*
- (B) *gayaki*, vocal
- (C) *baaj*, institution
- (D) *gharana*, lineage

Q.9 Two trains started at 7AM from the same point. The first train travelled north at a speed of 80km/h and the second train travelled south at a speed of 100 km/h. The time at which they were 540 km apart is _____ AM.

- (A) 9 (B) 10 (C) 11 (D) 11.30

Q.10 “I read somewhere that in ancient times the prestige of a kingdom depended upon the number of taxes that it was able to levy on its people. It was very much like the prestige of a head-hunter in his own community.”

Based on the paragraph above, the prestige of a head-hunter depended upon _____

- (A) the prestige of the kingdom
- (B) the prestige of the heads
- (C) the number of taxes he could levy
- (D) the number of heads he could gather

END OF THE QUESTION PAPER

Q. 1 – Q. 25 carry one mark each.

Q.1 For a balanced transportation problem with three sources and three destinations where costs, availabilities and demands are all finite and positive, which one of the following statements is **FALSE**?

- (A) The transportation problem does not have unbounded solution
- (B) The number of non-basic variables of the transportation problem is 4
- (C) The dual variables of the transportation problem are unrestricted in sign
- (D) The transportation problem has at most 5 basic feasible solutions

Q.2 Let $f : [a, b] \rightarrow \mathbb{R}$ (the set of all real numbers) be any function which is twice differentiable in (a, b) with only one root α in (a, b) . Let $f'(x)$ and $f''(x)$ denote the first and second order derivatives of $f(x)$ with respect to x . If α is a simple root and is computed by the Newton-Raphson method, then the method converges if

- (A) $|f(x)f''(x)| < |f'(x)|^2$, for all $x \in (a, b)$
- (B) $|f(x)f'(x)| < |f''(x)|$, for all $x \in (a, b)$
- (C) $|f'(x)f''(x)| < |f(x)|^2$, for all $x \in (a, b)$
- (D) $|f(x)f''(x)| < |f'(x)|$, for all $x \in (a, b)$

Q.3 Let $f : \mathbb{C} \rightarrow \mathbb{C}$ (the set of all complex numbers) be defined by

$$f(x + iy) = x^3 + 3xy^2 + i(y^3 + 3x^2y), \quad i = \sqrt{-1}.$$

Let $f'(z)$ denote the derivative of f with respect to z .

Then which one of the following statements is TRUE?

- (A) $f'(1+i)$ exists and $|f'(1+i)| = 3\sqrt{5}$
- (B) f is analytic at the origin
- (C) f is not differentiable at i
- (D) f is differentiable at 1

Q.4 The partial differential equation

$$(x^2 + y^2 - 1) \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + (x^2 + y^2 - 1) \frac{\partial^2 u}{\partial y^2} = 0$$

is

- (A) parabolic in the region $x^2 + y^2 > 2$
- (B) hyperbolic in the region $x^2 + y^2 > 2$
- (C) elliptic in the region $0 < x^2 + y^2 < 2$
- (D) hyperbolic in the region $0 < x^2 + y^2 < 2$

the set of all real numbers, and $\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$)

- (A) $\vec{\nabla} \phi$ is solenoidal but not irrotational
- (B) $\vec{\nabla} \phi$ is irrotational but not solenoidal
- (C) $\vec{\nabla} \phi$ is both solenoidal and irrotational
- (D) $\vec{\nabla} \phi$ is neither solenoidal nor irrotational

and $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$)

Which one of the following statements is TRUE?

- (A) f has a local maximum at origin
- (B) f has a local minimum at origin
- (C) f has a saddle point at origin
- (D) The origin is not a critical point of f

topology on \mathbb{R} .

and T_2 be the co-finite

Consider the following statements:

I. In (\mathbb{R}, T_1) , the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In (\mathbb{R}, T_2) , the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In (\mathbb{R}, T_1) , there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In (\mathbb{R}, T_2) , there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are TRUE?

(A) I and II only

(B) II and III only

(C) III and IV only

(D) I and IV only

Q.12 Let X and Y be normed linear spaces, and let $T : X \rightarrow Y$ be any bijective linear map with closed graph. Then which one of the following statements is TRUE?

(A) The graph of T is equal to $X \times Y$

(B) T^{-1} is continuous

(C) The graph of T^{-1} is closed

(D) T is continuous

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all real numbers).

If the constants a and b are such that

$$\frac{df}{dx} = a f(x) + 2 f(x+1) + b f(x+2), \text{ for all } x \in \mathbb{R},$$

then $4a + 3b$ is equal to _____ (round off to 2 places of decimal).



rational numbers) is _____.



is the set of all complex numbers).



$$\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$$

Which of the above statements is/are TRUE?

- (A) I and II only (B) I and IV only (C) IV only (D) III only

(the set of all complex numbers) such that

(i) for each $i \in \mathbb{N}$ (the set of all natural numbers), the i^{th} row $(k_{i,1}, k_{i,2}, \dots)$ of K is in ℓ^∞ and

(ii) for every $x = (x_1, x_2, \dots) \in \ell^1$, $\sum_{j=1}^{\infty} k_{i,j} x_j$ is summable for all $i \in \mathbb{N}$, and $(y_1, y_2, \dots) \in \ell^1$,

where $y_i = \sum_{j=1}^{\infty} k_{i,j} x_j$.

Let the set of all rows of K be denoted by E . Consider the following statements:

P: E is a bounded set in ℓ^∞ .

Q: E is a dense set in ℓ^∞ .

$$\left(\ell^1 = \{(x_1, x_2, \dots) : x_i \in \mathbb{C}, \sum_{i=1}^{\infty} |x_i| < \infty\} \right)$$

$$\left(\ell^\infty = \{(x_1, x_2, \dots) : x_i \in \mathbb{C}, \sup_{i \in \mathbb{N}} |x_i| < \infty\} \right)$$

Which of the above statements is/are TRUE?

- (A) Both P and Q (B) P only (C) Q only (D) Neither P nor Q

Q.29 Consider the following heat conduction problem for a finite rod

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - x e^t - 2t, \quad t > 0, \quad 0 < x < \pi,$$

with the boundary conditions $u(0, t) = -t^2$, $u(\pi, t) = -\pi e^t - t^2$, $t > 0$ and the initial condition $u(x, 0) = \sin x - \sin^3 x - x$, $0 \leq x \leq \pi$. If $v(x, t) = u(x, t) + x e^t + t^2$, then which one of the following is CORRECT?

- (A) $v(x, t) = \frac{1}{4} (e^{-t} \sin x + e^{-9t} \sin 3x)$
- (B) $v(x, t) = \frac{1}{4} (7e^{-t} \sin x - e^{-9t} \sin 3x)$
- (C) $v(x, t) = \frac{1}{4} (e^{-t} \sin x + e^{-3t} \sin 3x)$
- (D) $v(x, t) = \frac{1}{4} (3e^{-t} \sin x - e^{-3t} \sin 3x)$

denotes the additive group of rational numbers and $f: \mathbb{Q} \rightarrow \mathbb{Q}$ is a non-trivial homomorphism, then f is an isomorphism.

II. Any quotient group of a cyclic group is cyclic.

III. If every subgroup of a group G is a normal subgroup, then G is abelian.

IV. Every group of order 33 is cyclic.

Which of the above statements are TRUE?

(A) II and IV only

(B) II and III only

(C) I, II and IV only

(D) I, III and IV only



denotes the set of all integers, \mathbb{Z}_n denotes the set of all integers modulo n , for any positive integer n)

Which of the above statements are TRUE?

- (A) I, II and III only
- (C) I, II and IV only

- (B) I and III only
- (D) II and III only



(D) $\{g'_n\}_{n=1}^{\infty}$ converges uniformly on any closed interval which does **NOT** contain 0

numbers). Let a linear transformation $T: P_2 \rightarrow P_2$ be defined by

$$T(a+bx+cx^2) = (a+b) + (b-c)x + (a+c)x^2.$$

Consider the following statements:

- I. The null space of T is $\{\alpha(-1+x+x^2): \alpha \in \mathbb{R}\}$.
- II. The range space of T is spanned by the set $\{1+x^2, 1+x\}$.
- III. $T(T(1+x)) = 1+x^2$.
- IV. If M is the matrix representation of T with respect to the standard basis $\{1, x, x^2\}$ of P_2 , then the trace of the matrix M is 3.

Which of the above statements are TRUE?

- (A) I and II only
- (B) I, III and IV only
- (C) I, II and IV only
- (D) II and IV only

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)

Which of the above statements is/are TRUE?

(A) Both I and II

(B) I only

(C) II only

(D) Neither I nor II

Q.44 Consider the differential equation

$$t \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + t y = 0, \quad t > 0, \quad y(0+) = 1, \quad \left(\frac{dy}{dt} \right)_{t=0+} = 0.$$

If $Y(s)$ is the Laplace transform of $y(t)$, then the value of $Y(1)$ is _____ (round off to 2 places of decimal).

(Here, the inverse trigonometric functions assume principal values only)

Q.45 Let R be the region in the xy -plane bounded by the curves $y = x^2$, $y = 4x^2$, $xy = 1$ and $xy = 5$.

Then the value of the integral $\iint_R \frac{y^2}{x} dy dx$ is equal to _____.

Q.46 Let V be the vector space of all 3×3 matrices with complex entries over the real field. If

$$W_1 = \{A \in V : A = \bar{A}^T\} \quad \text{and} \quad W_2 = \{A \in V : \text{trace of } A = 0\},$$

then the dimension of $W_1 + W_2$ is equal to _____.

(\bar{A}^T denotes the conjugate transpose of A)

Q.47 The number of elements of order 15 in the additive group $\mathbb{Z}_{60} \times \mathbb{Z}_{50}$ is _____ .

(\mathbb{Z}_n denotes the group of integers modulo n , under the operation of addition modulo n , for any positive integer n)

Q.48 Consider the following cost matrix of assigning four jobs to four persons:

		Jobs			
		J ₁	J ₂	J ₃	J ₄
Persons	P ₁	5	8	6	10
	P ₂	2	5	4	8
	P ₃	6	7	6	9
	P ₄	6	9	8	10

Then the minimum cost of the assignment problem subject to the constraint that job J₄ is assigned to person P₂, is _____ .

Q.49 Let $y: [-1, 1] \rightarrow \mathbb{R}$ with $y(1)=1$ satisfy the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0 \text{ for } |x| < 1.$$

Then the value of $\int_{-1}^1 y(x)(x+x^2)dx$ is equal to _____ (round off to 2 places of decimal).

Q.50 Let \mathbb{Z}_{125} be the ring of integers modulo 125 under the operations of addition modulo 125 and multiplication modulo 125. If m is the number of maximal ideals of \mathbb{Z}_{125} and n is the number of non-units of \mathbb{Z}_{125} , then $m+n$ is equal to _____.

Q.51 The maximum value of the error term of the composite Trapezoidal rule when it is used to evaluate the definite integral

$$\int_{0.2}^{1.4} (\sin x - \log_e x) dx$$

with 12 sub-intervals of equal length, is equal to _____ (round off to 3 places of decimal).

Q.52 By the Simplex method, the optimal table of the linear programming problem:

$$\begin{aligned} &\text{Maximize } Z = \alpha x_1 + 3x_2 \\ &\text{subject to } \beta x_1 + x_2 + x_3 = 8, \\ &\quad \quad \quad 2x_1 + x_2 + x_4 = \gamma, \\ &\quad \quad \quad x_1, x_2, x_3, x_4 \geq 0, \end{aligned}$$

where α, β, γ are real constants, is

$c_j \rightarrow$	α	3	0	0	
Basic variable	x_1	x_2	x_3	x_4	Solution
x_2	1	0	2	-1	6
x_1	0	1	-1	1	2
$z_j - c_j$	0	0	2	1	-

Then the value of $\alpha + \beta + \gamma$ is _____.

Q.53 Consider the inner product space P_2 of all polynomials of degree at most 2 over the field of real numbers with the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ for $f, g \in P_2$.

Let $\{f_0, f_1, f_2\}$ be an orthogonal set in P_2 , where $f_0 = 1, f_1 = t + c_1, f_2 = t^2 + c_2 f_1 + c_3$ and c_1, c_2, c_3 are real constants. Then the value of $2c_1 + c_2 + 3c_3$ is equal to _____.

Q.54 Consider the system of linear differential equations

$$\frac{dx_1}{dt} = 5x_1 - 2x_2,$$

$$\frac{dx_2}{dt} = 4x_1 - x_2,$$

with the initial conditions $x_1(0) = 0, x_2(0) = 1$.

Then $\log_e(x_2(2) - x_1(2))$ is equal to _____.

Q.55 Consider the differential equation

$$x(1+x^2)\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 7y = 0.$$

The sum of the roots of the indicial equation of the Frobenius series solution for the above differential equation in a neighborhood of $x = 0$ is equal to _____.

END OF THE QUESTION PAPER