

**Notation**

$\mathbb{N}$	The set of all natural numbers $\{1,2,3, \dots\}$
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
$S_n$	The group of permutations of $n$ distinct symbols
$\mathbb{Z}_n$	$\{0, 1, 2, \dots, n - 1\}$ with addition and multiplication modulo $n$
$\phi$	empty set
$A^T$	Transpose of $A$
$i$	$\sqrt{-1}$
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive $x, y$ and $z$ axes of a three dimensional rectangular coordinate system
$\nabla$	$\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$
$I_n$	Identity matrix of order $n$
$\ln$	logarithm with base $e$

## SECTION – A

## MULTIPLE CHOICE QUESTIONS (MCQ)

**Q. 1 – Q.10 carry one mark each.**

Q.1 The sequence  $\{s_n\}$  of real numbers given by

$$s_n = \frac{\sin \frac{\pi}{2}}{1 \cdot 2} + \frac{\sin \frac{\pi}{2^2}}{2 \cdot 3} + \cdots + \frac{\sin \frac{\pi}{2^n}}{n \cdot (n+1)}$$

is

- (A) a divergent sequence
- (B) an oscillatory sequence
- (C) not a Cauchy sequence
- (D) a Cauchy sequence

Q.2 Let  $P$  be the vector space (over  $\mathbb{R}$ ) of all polynomials of degree  $\leq 3$  with real coefficients. Consider the linear transformation  $T: P \rightarrow P$  defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation  $M$  of  $T$  with respect to the ordered basis  $\{1, x, x^2, x^3\}$  satisfies

- (A)  $M^2 + I_4 = 0$
- (B)  $M^2 - I_4 = 0$
- (C)  $M - I_4 = 0$
- (D)  $M + I_4 = 0$

Q.3 Let  $f: [-1, 1] \rightarrow \mathbb{R}$  be a continuous function. Then the integral

$$\int_0^{\pi} x f(\sin x) dx$$

is equivalent to

- (A)  $\frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$
- (B)  $\frac{\pi}{2} \int_0^{\pi} f(\cos x) dx$
- (C)  $\pi \int_0^{\pi} f(\cos x) dx$
- (D)  $\pi \int_0^{\pi} f(\sin x) dx$

Q.4 Let  $\sigma$  be an element of the permutation group  $S_5$ . Then the maximum possible order of  $\sigma$  is

- (A) 5
- (B) 6
- (C) 10
- (D) 15

Q.5 Let  $f$  be a strictly monotonic continuous real valued function defined on  $[a, b]$  such that  $f(a) < a$  and  $f(b) > b$ . Then which one of the following is TRUE?

- (A) There exists exactly one  $c \in (a, b)$  such that  $f(c) = c$
- (B) There exist exactly two points  $c_1, c_2 \in (a, b)$  such that  $f(c_i) = c_i$ ,  $i = 1, 2$
- (C) There exists no  $c \in (a, b)$  such that  $f(c) = c$
- (D) There exist infinitely many points  $c \in (a, b)$  such that  $f(c) = c$

- Q.6 The value of  $\lim_{(x, y) \rightarrow (2, -2)} \frac{\sqrt{(x-y)-2}}{x-y-4}$  is  
 (A) 0 (B)  $\frac{1}{4}$  (C)  $\frac{1}{3}$  (D)  $\frac{1}{2}$
- Q.7 Let  $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$  and  $r = |\vec{r}|$ . If  $f(r) = \ln r$  and  $g(r) = \frac{1}{r}$ ,  $r \neq 0$ , satisfy  $2\nabla f + h(r)\nabla g = \vec{0}$ , then  $h(r)$  is  
 (A)  $r$  (B)  $\frac{1}{r}$  (C)  $2r$  (D)  $\frac{2}{r}$
- Q.8 The nonzero value of  $n$  for which the differential equation  

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0, \quad x \neq 0,$$
 becomes exact is  
 (A)  $-3$  (B)  $-2$  (C)  $2$  (D)  $3$
- Q.9 One of the points which lies on the solution curve of the differential equation  

$$(y - x)dx + (x + y)dy = 0,$$
 with the given condition  $y(0) = 1$ , is  
 (A)  $(1, -2)$  (B)  $(2, -1)$  (C)  $(2, 1)$  (D)  $(-1, 2)$
- Q.10 Let  $S$  be a closed subset of  $\mathbb{R}$ ,  $T$  a compact subset of  $\mathbb{R}$  such that  $S \cap T \neq \phi$ . Then  $S \cap T$  is  
 (A) closed but not compact  
 (B) not closed  
 (C) compact  
 (D) neither closed nor compact

**Q. 11 – Q. 30 carry two marks each.**

- Q.11 Let  $S$  be the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)2^{(2k-1)}}$$

and  $T$  be the series

$$\sum_{k=2}^{\infty} \left( \frac{3k-4}{3k+2} \right)^{\frac{(k+1)}{3}}$$

of real numbers. Then which one of the following is TRUE?

- (A) Both the series  $S$  and  $T$  are convergent  
 (B)  $S$  is convergent and  $T$  is divergent  
 (C)  $S$  is divergent and  $T$  is convergent  
 (D) Both the series  $S$  and  $T$  are divergent

Q.12 Let  $\{a_n\}$  be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \quad n \geq 1, \quad a_1 = 1.$$

Then all the terms of the sequence lie in

- (A)  $\left[\frac{1}{2}, \frac{3}{2}\right]$       (B)  $[0, 1]$       (C)  $[1, 2]$       (D)  $[1, 3]$

Q.13 The largest eigenvalue of the matrix  $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$  is

- (A) 16      (B) 21  
 (C) 48      (D) 64

Q.14 The value of the integral

$$\frac{(2n)!}{2^{2n} (n!)^2} \int_{-1}^1 (1-x^2)^n dx, \quad n \in \mathbb{N}$$

is

- (A)  $\frac{2}{(2n+1)!}$       (B)  $\frac{2n}{(2n+1)!}$   
 (C)  $\frac{2(n!)}{2n+1}$       (D)  $\frac{(n+1)!}{2n+1}$

Q.15 If the triple integral over the region bounded by the planes

$$2x + y + z = 4, \quad x = 0, \quad y = 0, \quad z = 0$$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz dy dx,$$

then the function  $\lambda(x) - \mu(x, y)$  is

- (A)  $x + y$       (B)  $x - y$       (C)  $x$       (D)  $y$

Q.16 The surface area of the portion of the plane  $y + 2z = 2$  within the cylinder  $x^2 + y^2 = 3$  is

- (A)  $\frac{3\sqrt{5}}{2}\pi$       (B)  $\frac{5\sqrt{5}}{2}\pi$       (C)  $\frac{7\sqrt{5}}{2}\pi$       (D)  $\frac{9\sqrt{5}}{2}\pi$

Q.17 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} \frac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}.$$

Then the value of  $\left(\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y \partial x}\right)$  at the point  $(0, 0)$  is

- (A) 0                      (B) 1                      (C) 2                      (D) 4

Q.18 The function  $f(x, y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$  has a saddle point at

- (A)  $(0, 0)$                       (B)  $(0, 2)$                       (C)  $(1, 1)$                       (D)  $(-2, 1)$

Q.19 Consider the vector field  $\vec{F} = r^\beta(y\hat{i} - x\hat{j})$ , where  $\beta \in \mathbb{R}$ ,  $\vec{r} = x\hat{i} + y\hat{j}$  and  $r = |\vec{r}|$ . If the absolute value of the line integral  $\oint_C \vec{F} \cdot d\vec{r}$  along the closed curve  $C: x^2 + y^2 = a^2$  (oriented counter clockwise) is  $2\pi$ , then  $\beta$  is

- (A)  $-2$                       (B)  $-1$                       (C) 1                      (D) 2

Q.20 Let  $S$  be the surface of the cone  $z = \sqrt{x^2 + y^2}$  bounded by the planes  $z = 0$  and  $z = 3$ . Further, let  $C$  be the closed curve forming the boundary of the surface  $S$ . A vector field  $\vec{F}$  is such that  $\nabla \times \vec{F} = -x\hat{i} - y\hat{j}$ . The absolute value of the line integral  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $r = |\vec{r}|$ , is

- (A) 0                      (B)  $9\pi$                       (C)  $15\pi$                       (D)  $18\pi$

Q.21 Let  $y(x)$  be the solution of the differential equation

$$\frac{d}{dx}\left(x \frac{dy}{dx}\right) = x; \quad y(1) = 0, \quad \left.\frac{dy}{dx}\right|_{x=1} = 0.$$

Then  $y(2)$  is

- (A)  $\frac{3}{4} + \frac{1}{2} \ln 2$                       (B)  $\frac{3}{4} - \frac{1}{2} \ln 2$   
 (C)  $\frac{3}{4} + \ln 2$                       (D)  $\frac{3}{4} - \ln 2$

Q.22 The general solution of the differential equation with constant coefficients

$$\frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

approaches zero as  $x \rightarrow \infty$ , if

- (A)  $b$  is negative and  $c$  is positive  
 (B)  $b$  is positive and  $c$  is negative  
 (C) both  $b$  and  $c$  are positive  
 (D) both  $b$  and  $c$  are negative

Q.23 Let  $S \subset \mathbb{R}$  and  $\partial S$  denote the set of points  $x$  in  $\mathbb{R}$  such that every neighbourhood of  $x$  contains some points of  $S$  as well as some points of complement of  $S$ . Further, let  $\bar{S}$  denote the closure of  $S$ . Then which one of the following is FALSE?

- (A)  $\partial \mathbb{Q} = \mathbb{R}$
- (B)  $\partial(\mathbb{R} \setminus T) = \partial T, \quad T \subset \mathbb{R}$
- (C)  $\partial(T \cup V) = \partial T \cup \partial V, \quad T, V \subset \mathbb{R}, T \cap V \neq \emptyset$
- (D)  $\partial T = \bar{T} \cap (\mathbb{R} \setminus \bar{T}), \quad T \subset \mathbb{R}$

Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$$

is

- (A)  $\frac{1}{3} \ln 2 - \frac{5}{18}$
- (B)  $\frac{1}{3} \ln 2 - \frac{5}{6}$
- (C)  $\frac{2}{3} \ln 2 - \frac{5}{18}$
- (D)  $\frac{2}{3} \ln 2 - \frac{5}{6}$

Q.25 Let  $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$  for all  $x \in [-1, 1]$ . Then which one of the following is TRUE?

- (A) Maximum value of  $f(x)$  is  $\frac{3}{2}$
- (B) Minimum value of  $f(x)$  is  $\frac{1}{3}$
- (C) Maximum of  $f(x)$  occurs at  $x = \frac{1}{2}$
- (D) Minimum of  $f(x)$  occurs at  $x = 1$

Q.26 The matrix  $M = \begin{bmatrix} \cos \alpha & \sin \alpha \\ i \sin \alpha & i \cos \alpha \end{bmatrix}$  is a unitary matrix when  $\alpha$  is

- (A)  $(2n + 1) \frac{\pi}{2}, n \in \mathbb{Z}$
- (B)  $(3n + 1) \frac{\pi}{3}, n \in \mathbb{Z}$
- (C)  $(4n + 1) \frac{\pi}{4}, n \in \mathbb{Z}$
- (D)  $(5n + 1) \frac{\pi}{5}, n \in \mathbb{Z}$

Q.27 Let  $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}, \alpha \in \mathbb{R} \setminus \{0\}$  and  $\mathbf{b}$  a non-zero vector such that  $M\mathbf{x} = \mathbf{b}$  for some  $\mathbf{x} \in \mathbb{R}^3$ . Then the value of  $\mathbf{x}^T \mathbf{b}$  is

- (A)  $-\alpha$
- (B)  $\alpha$
- (C) 0
- (D) 1

Q.28 The number of group homomorphisms from the cyclic group  $\mathbb{Z}_4$  to the cyclic group  $\mathbb{Z}_7$  is

- (A) 7
- (B) 3
- (C) 2
- (D) 1

Q.29 In the permutation group  $S_n (n \geq 5)$ , if  $H$  is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

- (A) Order of  $H$  is 2
- (B) Index of  $H$  in  $S_n$  is 2
- (C)  $H$  is abelian
- (D)  $H = S_n$

Q.30 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as

$$f(x) = \begin{cases} x(1 + x^\alpha \sin(\ln x^2)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then, at  $x = 0$ , the function  $f$  is

- (A) continuous and differentiable when  $\alpha = 0$
- (B) continuous and differentiable when  $\alpha > 0$
- (C) continuous and differentiable when  $-1 < \alpha < 0$
- (D) continuous and differentiable when  $\alpha < -1$

### SECTION - B

#### MULTIPLE SELECT QUESTIONS (MSQ)

**Q. 31 – Q. 40 carry two marks each.**

Q.31 Let  $\{s_n\}$  be a sequence of positive real numbers satisfying

$$2s_{n+1} = s_n^2 + \frac{3}{4}, \quad n \geq 1.$$

If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 2x + \frac{3}{4} = 0$  and  $\alpha < s_1 < \beta$ , then which of the following statement(s) is(are) TRUE ?

- (A)  $\{s_n\}$  is monotonically decreasing
- (B)  $\{s_n\}$  is monotonically increasing
- (C)  $\lim_{n \rightarrow \infty} s_n = \alpha$
- (D)  $\lim_{n \rightarrow \infty} s_n = \beta$

Q.32 The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx, \quad n \geq 1$$

is (are)

- (A) 0 when  $n$  is even
- (B) 0 when  $n$  is odd
- (C)  $-\frac{4}{n^2}$  when  $n$  is even
- (D)  $-\frac{4}{n^2}$  when  $n$  is odd

Q.33 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} \frac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Then at the point  $(0, 0)$ , which of the following statement(s) is(are) TRUE ?

- (A)  $f$  is not continuous
- (B)  $f$  is continuous
- (C)  $f$  is differentiable
- (D) Both first order partial derivatives of  $f$  exist

Q.34 Consider the vector field  $\vec{F} = x\hat{i} + y\hat{j}$  on an open connected set  $S \subset \mathbb{R}^2$ . Then which of the following statement(s) is(are) TRUE ?

- (A) Divergence of  $\vec{F}$  is zero on  $S$
- (B) The line integral of  $\vec{F}$  is independent of path in  $S$
- (C)  $\vec{F}$  can be expressed as a gradient of a scalar function on  $S$
- (D) The line integral of  $\vec{F}$  is zero around any piecewise smooth closed path in  $S$

Q.35 Consider the differential equation

$$\sin 2x \frac{dy}{dx} = 2y + 2 \cos x, \quad y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

- (A) The solution is unbounded when  $x \rightarrow 0$
- (B) The solution is unbounded when  $x \rightarrow \frac{\pi}{2}$
- (C) The solution is bounded when  $x \rightarrow 0$
- (D) The solution is bounded when  $x \rightarrow \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

- (A) There exists a connected set in  $\mathbb{R}$  which is not compact
- (B) Arbitrary union of closed intervals in  $\mathbb{R}$  need not be compact
- (C) Arbitrary union of closed intervals in  $\mathbb{R}$  is always closed
- (D) Every bounded infinite subset  $V$  of  $\mathbb{R}$  has a limit point in  $V$  itself

Q.37 Let  $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$  for all  $x \in \mathbb{R}$ . Then which of the following statement(s) is(are) TRUE?

- (A) The equation  $P(x) = 0$  has exactly one solution in  $\mathbb{R}$
- (B)  $P(x)$  is strictly increasing for all  $x \in \mathbb{R}$
- (C) The equation  $P(x) = 0$  has exactly two solutions in  $\mathbb{R}$
- (D)  $P(x)$  is strictly decreasing for all  $x \in \mathbb{R}$

- Q.38 Let  $G$  be a finite group and  $o(G)$  denotes its order. Then which of the following statement(s) is(are) TRUE?
- (A)  $G$  is abelian if  $o(G) = pq$  where  $p$  and  $q$  are distinct primes  
 (B)  $G$  is abelian if every non identity element of  $G$  is of order 2  
 (C)  $G$  is abelian if the quotient group  $\frac{G}{Z(G)}$  is cyclic, where  $Z(G)$  is the center of  $G$   
 (D)  $G$  is abelian if  $o(G) = p^3$ , where  $p$  is prime

- Q.39 Consider the set  $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid \alpha x + \beta y + z = \gamma, \alpha, \beta, \gamma \in \mathbb{R} \right\}$ . For which of the following choice(s) the set  $V$  becomes a two dimensional subspace of  $\mathbb{R}^3$  over  $\mathbb{R}$  ?
- (A)  $\alpha = 0, \beta = 1, \gamma = 0$   
 (B)  $\alpha = 0, \beta = 1, \gamma = 1$   
 (C)  $\alpha = 1, \beta = 0, \gamma = 0$   
 (D)  $\alpha = 1, \beta = 1, \gamma = 0$

- Q.40 Let  $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \mid n, m \in \mathbb{N} \right\}$ . Then which of the following statement(s) is(are) TRUE?
- (A)  $S$  is closed  
 (B)  $S$  is not open  
 (C)  $S$  is connected  
 (D)  $0$  is a limit point of  $S$

### SECTION – C

#### NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

- Q.41 Let  $\{s_n\}$  be a sequence of real numbers given by

$$s_n = 2^{(-1)^n} \left( 1 - \frac{1}{n} \right) \sin \frac{n\pi}{2}, \quad n \in \mathbb{N}.$$

Then the least upper bound of the sequence  $\{s_n\}$  is \_\_\_\_\_

- Q.42 Let  $\{s_k\}$  be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \quad k \geq 1, \quad \alpha > 0.$$

Then

$$\lim_{n \rightarrow \infty} (s_1 s_2 \dots s_n)^{1/n}$$

is \_\_\_\_\_

- Q.43 Let  $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$  be a non-zero vector and  $A = \frac{\mathbf{x}\mathbf{x}^T}{\mathbf{x}^T\mathbf{x}}$ . Then the dimension of the vector space  $\{\mathbf{y} \in \mathbb{R}^3 \mid A\mathbf{y} = \mathbf{0}\}$  over  $\mathbb{R}$  is \_\_\_\_\_

- Q.44 Let  $f$  be a real valued function defined by

$$f(x, y) = 2 \ln(x^2 y^2 e^{\frac{y}{x}}), \quad x > 0, y > 0.$$

Then the value of  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$  at any point  $(x, y)$ , where  $x > 0, y > 0$ , is \_\_\_\_\_

- Q.45 Let  $\vec{F} = \sqrt{x} \hat{i} + (x + y^3) \hat{j}$  be a vector field for all  $(x, y)$  with  $x \geq 0$  and  $\vec{r} = x \hat{i} + y \hat{j}$ . Then the value of the line integral  $\int_C \vec{F} \cdot d\vec{r}$  from  $(0, 0)$  to  $(1, 1)$  along the path  $C: x = t^2, y = t^3, 0 \leq t \leq 1$  is \_\_\_\_\_

- Q.46 If  $f: (-1, \infty) \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{x}{1+x}$  is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9}(x-2) + \frac{c(x-2)^2}{(1+\xi)^3},$$

where  $\xi$  lies between 2 and  $x$ , then the value of  $c$  is \_\_\_\_\_

- Q.47 Let  $y_1(x), y_2(x)$  and  $y_3(x)$  be linearly independent solutions of the differential equation

$$\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0.$$

If the Wronskian  $W(y_1, y_2, y_3)$  is of the form  $ke^{bx}$  for some constant  $k$ , then the value of  $b$  is \_\_\_\_\_

- Q.48 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)} (x+2)^{2n} \text{ is } \underline{\hspace{2cm}}$$

Q.49 Let  $f: (0, \infty) \rightarrow \mathbb{R}$  be a continuous function such that

$$\int_0^x f(t) dt = -2 + \frac{x^2}{2} + 4x \sin 2x + 2 \cos 2x.$$

Then the value of  $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$  is \_\_\_\_\_

Q.50 Let  $G$  be a cyclic group of order 12. Then the number of non-isomorphic subgroups of  $G$  is \_\_\_\_\_

**Q. 51 – Q. 60 carry two marks each.**

Q.51 The value of  $\lim_{n \rightarrow \infty} \left(8n - \frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$  is equal to \_\_\_\_\_

Q.52 Let  $R$  be the region enclosed by  $x^2 + 4y^2 \geq 1$  and  $x^2 + y^2 \leq 1$ . Then the value of

$$\iint_R |xy| dx dy \quad \text{is _____}$$

Q.53 Let

$$M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix}, \quad \alpha\beta\gamma = 1, \quad \alpha, \beta, \gamma \in \mathbb{R} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.$$

Then  $M\mathbf{x} = \mathbf{0}$  has infinitely many solutions if  $\text{trace}(M)$  is \_\_\_\_\_

Q.54 Let  $C$  be the boundary of the region enclosed by  $y = x^2$ ,  $y = x + 2$ , and  $x = 0$ . Then the value of the line integral

$$\oint_C (xy - y^2) dx - x^3 dy,$$

where  $C$  is traversed in the counter clockwise direction, is \_\_\_\_\_

- Q.55 Let  $S$  be the closed surface forming the boundary of the region  $V$  bounded by  $x^2 + y^2 = 3$ ,  $z = 0$ ,  $z = 6$ . A vector field  $\vec{F}$  is defined over  $V$  with  $\nabla \cdot \vec{F} = 2y + z + 1$ . Then the value of

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n} \, ds,$$

where  $\hat{n}$  is the unit outward drawn normal to the surface  $S$ , is \_\_\_\_\_,

- Q.56 Let  $y(x)$  be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0, \quad y(0) = 1, \quad \left. \frac{dy}{dx} \right|_{x=0} = -1.$$

Then  $y(x)$  attains its maximum value at  $x =$  \_\_\_\_\_

- Q.57 The value of the double integral

$$\int_0^{\pi} \int_0^x \frac{\sin y}{\pi - y} \, dy \, dx$$

is \_\_\_\_\_

- Q.58 Let  $H$  denote the group of all  $2 \times 2$  invertible matrices over  $\mathbb{Z}_5$  under usual matrix multiplication. Then the order of the matrix  $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  in  $H$  is \_\_\_\_\_

- Q.59 Let  $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$ ,  $N(A)$  the null space of  $A$  and  $R(B)$  the range space of  $B$ .

Then the dimension of  $N(A) \cap R(B)$  over  $\mathbb{R}$  is \_\_\_\_\_

- Q.60 The maximum value of  $f(x, y) = x^2 + 2y^2$  subject to the constraint  $y - x^2 + 1 = 0$  is \_\_\_\_\_

**END OF THE QUESTION PAPER**

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	D	1
2	MCQ	B	1
3	MCQ	A	1
4	MCQ	B	1
5	MCQ	A	1
6	MCQ	B	1
7	MCQ	C	1
8	MCQ	D	1
9	MCQ	C	1
10	MCQ	C	1
11	MCQ	B	2
12	MCQ	D	2
13	MCQ	B	2
14	MCQ	C	2
15	MCQ	D	2
16	MCQ	A	2
17	MCQ	B	2
18	MCQ	D	2
19	MCQ	A	2
20	MCQ	MTA	2
21	MCQ	B	2
22	MCQ	C	2
23	MCQ	C	2
24	MCQ	C	2
25	MCQ	A	2
26	MCQ	A	2
27	MCQ	C	2
28	MCQ	D	2
29	MCQ	B	2
30	MCQ	MTA	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;C	2
32	MSQ	A;D	2
33	MSQ	B;D	2
34	MSQ	B;C;D	2
35	MSQ	C;D	2
36	MSQ	A;B	2
37	MSQ	A;D	2
38	MSQ	B;C	2
39	MSQ	A;C;D	2
40	MSQ	B;D	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	0.5:0.5	1
42	NAT	1.0:1.0	1
43	NAT	2.0:2.0	1
44	NAT	8.0:8.0	1
45	NAT	1.49:1.55	1
46	NAT	-1:-1	1
47	NAT	6.0:6.0	1
48	NAT	0.5:0.5	1
49	NAT	0.25:0.25	1
50	NAT	6.0:6.0	1
51	NAT	1.0:1.0	2
52	NAT	0.35:0.4	2
53	NAT	3.0:3.0	2
54	NAT	0.8:1.9	2
55	NAT	72.0:72.0	2
56	NAT	-0.3:-0.25	2
57	NAT	2.0:2.0	2
58	NAT	3.0:3.0	2
59	NAT	1.0:1.0	2
60	NAT	2.0:2.0	2