

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following functions has a discontinuity in the second derivative at $x = 0$, where x is a real variable?

- (A) $f(x) = |x|^3$
- (B) $f(x) = x|x|$
- (C) $f(x) = \cos(|x|)$
- (D) $f(x) = |x|^2$

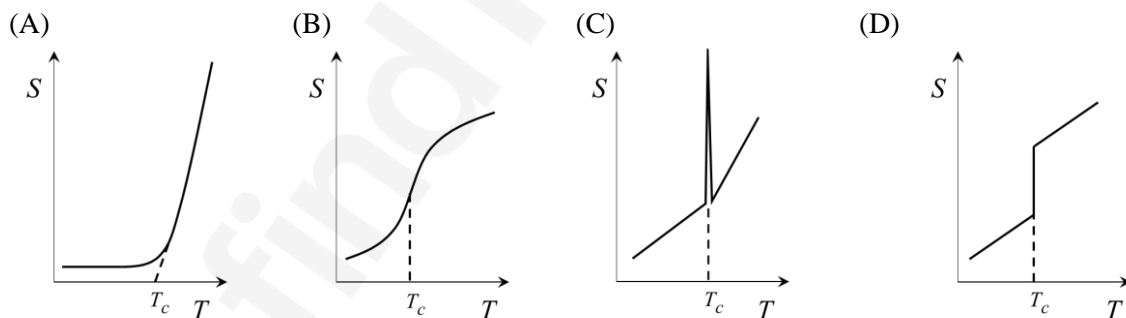
Q.2 A collimated beam of laser light of wavelength 514 nm is normally incident on a smooth glass slab placed in air. Given the refractive indices of glass and air are 1.47 and 1.0, respectively, the percentage of light intensity reflected back is

- (A) 0
- (B) 4.0
- (C) 3.6
- (D) 4.2

Q.3 Two stationary point particles with equal and opposite charges are at some fixed distance from each other. The points having zero electric potential lie on:

- (A) A sphere
- (B) A plane
- (C) A cylinder
- (D) Two parallel planes

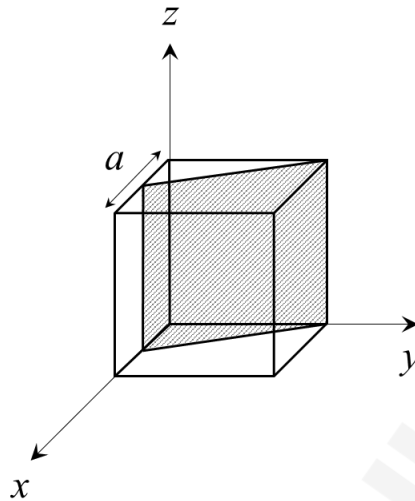
Q.4 For a system undergoing a first order phase transition at a temperature T_c , which one of the following graphs best describes the variation of entropy (S) as a function of temperature (T)?



Q.5 In a photoelectric effect experiment, a monochromatic light source emitting photons with energy greater than the work function of the metal under test is used. If the power of the light source is doubled, which one of the following statements is correct?

- (A) The number of emitted photoelectrons remains the same.
- (B) The stopping potential remains the same.
- (C) The number of emitted photoelectrons decreases.
- (D) The stopping potential doubles.

- Q.6 The figure below shows a cubic unit cell with lattice constant a . The shaded crystallographic plane intersects the x -axis at $0.5a$. The Miller indices of the shaded plane are

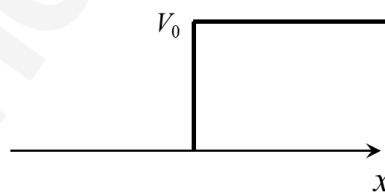


- (A) (210) (B) $(\bar{2}10)$ (C) (110) (D) (102)

- Q.7 For a particle moving in a central potential, which one of the following statements is correct?

- (A) The motion is restricted to a plane due to the conservation of angular momentum.
 (B) The motion is restricted to a plane due to the conservation of energy only.
 (C) The motion is restricted to a plane due to the conservation of linear momentum.
 (D) The motion is not restricted to a plane.

- Q.8 Consider the motion of a quantum particle of mass m and energy E under the influence of a step potential of height V_0 . If R denotes the reflection coefficient, which one of the following statements is true?



- (A) If $E = \frac{4}{3}V_0$, $R = 1$ (B) If $E = \frac{4}{3}V_0$, $R = 0$
 (C) If $E = \frac{1}{2}V_0$, $R = 1$ (D) If $E = \frac{1}{2}V_0$, $R = 0.5$

- Q.9 The Boolean function $\bar{P}\bar{Q}(\bar{P} + Q)(Q + \bar{Q})$ is equivalent to:

- (A) P (B) \bar{P} (C) $\bar{P}Q$ (D) $P\bar{Q}$

Q.10 Three point charges each carrying a charge q are placed on the vertices of an equilateral triangle of side L . The electrostatic potential energy of the configuration is:

- (A) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{L}$ (B) $\frac{2}{4\pi\epsilon_0} \frac{q^2}{L}$
 (C) $\frac{3}{4\pi\epsilon_0} \frac{q^2}{L}$ (D) $\frac{1}{\pi\epsilon_0} \frac{q^2}{L}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Which one of the following statements is correct?

Given, $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ is the binomial coefficient.

- (A) $\cos n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta - \dots$
 (B) $\sin n\theta = \binom{n}{1} \cos^{n-1} \theta \sin \theta + \binom{n}{3} \cos^{n-3} \theta \sin^3 \theta + \dots$
 (C) $\cos n\theta = \cos^n \theta + \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta + \dots$
 (D) $\sin n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta - \dots$

Q.12

The volume integral $\int_V e^{-\left(\frac{r}{R}\right)^2} \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) d^3r$, where V is the volume of a sphere of radius R centered at the origin, is equal to

- (A) 4π (B) 0 (C) $\frac{4}{3} \pi R^3$ (D) 1

Q.13 $\lim_{x \rightarrow 0^+} x^x$ is equal to

- (A) 0 (B) ∞ (C) e (D) 1

Q.14 A wheel is rotating at a frequency f_0 Hz about a fixed vertical axis. The wheel stops in t_0 seconds, with constant angular deceleration. The number of turns covered by the wheel before it comes to rest is given by:

- (A) $f_0 t_0$ (B) $2f_0 t_0$ (C) $f_0 t_0 / 2$ (D) $f_0 t_0 / \sqrt{2}$

Q.15 Two objects of masses m and $2m$ are moving at speeds of v and $v/2$, respectively. After undergoing a completely inelastic collision, they move together with a speed of $v/3$. The angle between the initial velocity vectors of the two objects is

- (A) 60° (B) 120° (C) 45° (D) 90°

Q.16 Two planets P_1 and P_2 having masses M_1 and M_2 revolve around the Sun in elliptical orbits, with time periods T_1 and T_2 , respectively. The minimum and maximum distances of planet P_1 from the Sun are R and $3R$, respectively, whereas for planet P_2 these are $2R$ and $4R$, respectively, where R is a constant. Assuming M_1 and M_2 are much smaller than the mass of the Sun, the magnitude of $\frac{T_2}{T_1}$ is

- (A) $\frac{2}{3} \sqrt{\frac{2M_1}{3M_2}}$ (B) $\frac{3}{2} \sqrt{\frac{3M_2}{2M_1}}$ (C) $\frac{3}{2} \sqrt{\frac{3}{2}}$ (D) $\frac{2}{3} \sqrt{\frac{2}{3}}$

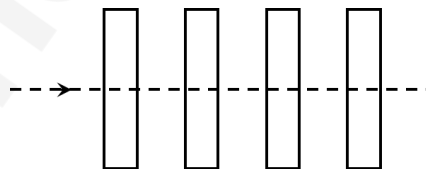
Q.17 The intensity of the primary maximum in a two-slit interference pattern is given by I_2 and the intensity of the primary maximum in a three-slit interference pattern is given by I_3 . Assuming the far-field approximation, same slit parameters and intensity of the incident light in both the cases, I_2 and I_3 are related as

- (A) $I_2 = \frac{3}{2} I_3$ (B) $I_2 = \frac{9}{4} I_3$ (C) $I_2 = \frac{2}{3} I_3$ (D) $I_2 = \frac{4}{9} I_3$

Q.18 A short rod of length L and negligible diameter lies along the optical axis of a concave mirror at a distance of 3 m. The focal length of the mirror is 1 m and $L \ll 1$ m. If L' is the length of image of the object in the mirror, then

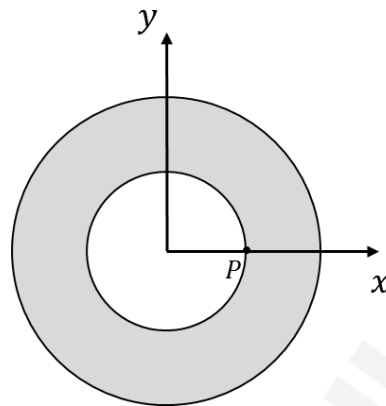
- (A) $\frac{L'}{L} = 4$ (B) $\frac{L'}{L} = 2$ (C) $\frac{L'}{L} = \frac{1}{16}$ (D) $\frac{L'}{L} = \frac{1}{4}$

Q.19 A beam of unpolarized light of intensity I_0 falls on a system of four identical linear polarizers placed in a line as shown in the figure. The transmission axes of any two successive polarizers make an angle of 30° with each other. If the transmitted light has intensity I , the ratio $\frac{I}{I_0}$ is



- (A) $\frac{81}{256}$ (B) $\frac{9}{16}$ (C) $\frac{27}{64}$ (D) $\frac{27}{128}$

- Q.20 Consider an annular region in free space containing a uniform magnetic field in the z -direction, schematically represented by the shaded region in the figure. A particle having charge Q and mass M starts off from point $P (a, 0, 0)$ in the $+x$ -direction with constant speed v . If the radii of inner and outer circles are a and b , respectively, the minimum magnetic field required so that the particle returns to the inner circle is



- (A) $\frac{Mv}{Q} \left(\frac{b^2-a^2}{b}\right)^{-1}$ (B) $\frac{Mv}{Q} \left(\frac{b^2-a^2}{2b}\right)^{-1}$
 (C) $\frac{Mv}{Q} \left(\frac{b^2-a^2}{3b}\right)^{-1}$ (D) $\frac{Mv}{Q} \left(\frac{b^2-a^2}{4b}\right)^{-1}$
- Q.21 A thin conducting square loop of side L is placed in the first quadrant of the xy -plane with one of the vertices at the origin. If a changing magnetic field $\vec{B}(t) = \beta_0(5zyt \hat{x} + zxt \hat{y} + 3y^2t \hat{z})$ is applied, where β_0 is a constant, then the magnitude of the induced electromotive force in the loop is

- (A) $4\beta_0 L^4$ (B) $3\beta_0 L^4$ (C) $2\beta_0 L^4$ (D) $\beta_0 L^4$

- Q.22 In which one of the following limits the Fermi-Dirac distribution $n_F(\epsilon, T) = \left(e^{\frac{\epsilon-\mu}{k_B T}} + 1\right)^{-1}$ and Bose-Einstein distribution $n_B(\epsilon, T) = \left(e^{\frac{\epsilon-\mu}{k_B T}} - 1\right)^{-1}$ reduce to Maxwell-Boltzmann distribution? (Here ϵ is the energy of the state, μ is the chemical potential, k_B is the Boltzmann constant and T is the temperature).

- (A) $\mu = 0$ (B) $(\epsilon - \mu) \ll k_B T$
 (C) $(\epsilon - \mu) \gg k_B T$ (D) $\mu \gg k_B T$

- Q.23 Consider N classical particles at temperature T , each of which can have two possible energies 0 and ϵ . The number of particles in the lower energy level (N_0) and higher energy level (N_ϵ) levels are related by (k_B is the Boltzmann constant)

- (A) $\frac{N_0}{N_\epsilon} = e^{\frac{-\epsilon}{k_B T}}$ (B) $\frac{N_0}{N_\epsilon} = e^{\frac{\epsilon}{k_B T}}$
 (C) $\frac{N_0}{N_\epsilon} = 1 + e^{\frac{\epsilon}{k_B T}}$ (D) $\frac{N_0}{N_\epsilon} = 1 - e^{\frac{-\epsilon}{k_B T}}$

Q.24 The root mean square (*rms*) speeds of Hydrogen atoms at 500 K, V_H , and Helium atoms at 2000 K, V_{He} , are related as

- (A) $V_H > V_{He}$ (B) $V_H < V_{He}$ (C) $V_H = V_{He}$ (D) $V_H \gg V_{He}$

Q.25 The normalized ground-state wave function of a one-dimensional quantum harmonic oscillator with force constant K and mass m is $\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, where $\alpha = m\omega_0/\hbar$ and $\omega_0^2 = K/m$. Which one of the following is the probability of finding the particle outside the classically allowed region?

(The classically allowed region is where the total energy is greater than the potential energy)

- (A) $\frac{2}{\sqrt{\pi}} \int_1^\infty y^2 e^{-y^2} dy$ (B) $\frac{2}{\sqrt{\pi}} \int_1^\infty e^{-y^2} dy$
 (C) 0.5 (D) 0

Q.26 A linear operator \hat{O} acts on two orthonormal states of a system ψ_1 and ψ_2 as per following:

$\hat{O}\psi_1 = \psi_2$, $\hat{O}\psi_2 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. The system is in a superposed state defined by

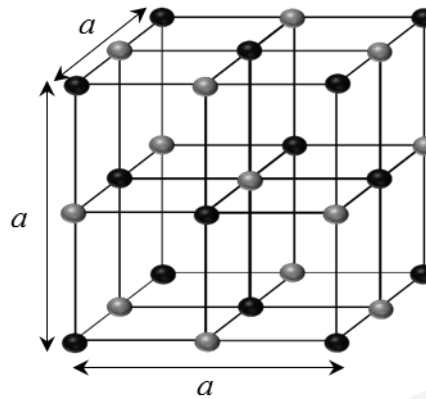
$\psi = \frac{1}{\sqrt{2}}\psi_1 + \frac{i}{\sqrt{2}}\psi_2$. The expectation value of \hat{O} in the state ψ is

- (A) $\frac{1}{2\sqrt{2}}(1 + i(\sqrt{2} + 1))$ (B) $\frac{1}{2\sqrt{2}}(1 - i(\sqrt{2} + 1))$
 (C) $\frac{1}{2\sqrt{2}}(1 + i(\sqrt{2} - 1))$ (D) $\frac{1}{2\sqrt{2}}(1 - i(\sqrt{2} - 1))$

Q.27 Consider a one-dimensional infinite potential well of width a . This system contains five non-interacting electrons, each of mass m , at temperature $T = 0$ K. The energy of the highest occupied state is

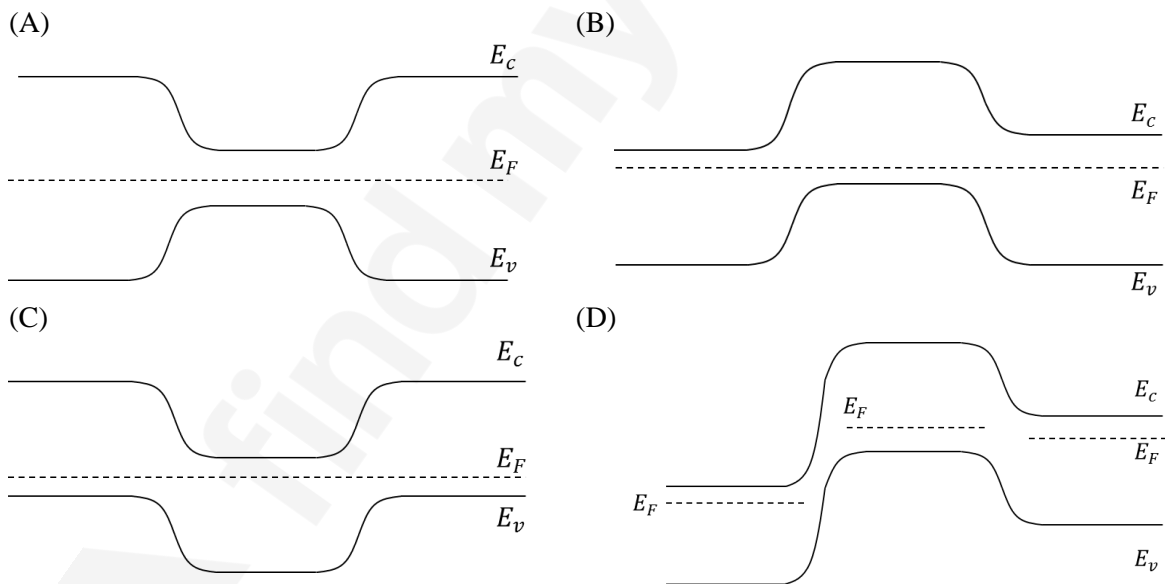
- (A) $\frac{25\pi^2 \hbar^2}{2ma^2}$ (B) $\frac{10\pi^2 \hbar^2}{2ma^2}$
 (C) $\frac{5\pi^2 \hbar^2}{2ma^2}$ (D) $\frac{9\pi^2 \hbar^2}{2ma^2}$

Q.28 Consider the crystal structure shown in the figure, where black and grey spheres represent atoms of two different elements and a denotes the lattice constant. The Bravais lattice for this structure is



- (A) Simple cubic
- (B) Face-centered cubic
- (C) Body-centered cubic
- (D) Triclinic

Q.29 For an unbiased Silicon $n-p-n$ transistor in thermal equilibrium, which one of the following electronic energy band diagrams is correct? (E_c = conduction band minimum, E_v = valence band maximum, E_F = Fermi level).



- Q.33 A thin rod of uniform density and length $2\sqrt{3}$ m is undergoing small oscillations about a pivot point. The time period of oscillation (T_m) is minimum when the distance of the pivot point from the center-of-mass of the rod is x_m . Which of the following is/are correct?
(Assume acceleration due to gravity $g = 10 \text{ m/s}^2$)
- (A) $x_m = 1 \text{ m}$ (B) $x_m = \frac{\sqrt{3}}{2} \text{ m}$ (C) $T_m = \frac{2\pi}{\sqrt{3}} \text{ s}$ (D) $T_m = \frac{2\pi}{\sqrt{5}} \text{ s}$
- Q.34 Three sinusoidal waves of the same frequency travel with the same speed along the positive x -direction. The amplitudes of the waves are a , $a/2$, and $a/3$, and the phase constants of the waves are $\pi/2$, π , and $3\pi/2$, respectively. If A_m and φ_m are the amplitude and phase constant of the wave resulting from the superposition of the three waves, which of the following is/are correct?
- (A) $A_m = \frac{5}{6} a$ (B) $\varphi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$
(C) $A_m = \frac{7}{6} a$ (D) $\varphi_m = \tan^{-1}\left(\frac{2}{3}\right)$
- Q.35 An object executes simple harmonic motion along the x -direction with angular frequency ω and amplitude a . The speed of the object is 4 cm/s and 2 cm/s, when it is at distances 2 cm and 6 cm, respectively from the equilibrium position. Which of the following is/are correct?
- (A) $\omega = \sqrt{\frac{3}{8}} \text{ rad/s}$ (B) $\omega = \sqrt{\frac{5}{6}} \text{ rad/s}$
(C) $a = \sqrt{\frac{140}{3}} \text{ cm}$ (D) $a = \sqrt{\frac{175}{6}} \text{ cm}$
- Q.36 For electric and magnetic fields, \vec{E} and \vec{B} , due to a charge density $\rho(\vec{r}, t)$ and a current density $\vec{J}(\vec{r}, t)$, which of the following relations is/are always correct?
- (A) $\vec{\nabla} \times \vec{E} = 0$
(B) $\vec{\nabla} \cdot \vec{B} = 0$
(C) $\vec{\nabla} \cdot \vec{J} - \frac{\partial \rho}{\partial t} = 0$
(D) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where \vec{F} is the force on a particle with charge q moving with velocity \vec{v}

Q.37 A spherical dielectric shell with inner radius a and outer radius b , has polarization $\vec{P} = \frac{k}{r^2} \hat{r}$, where k is a constant and \hat{r} is the unit vector along the radial direction. Which of the following statements is/are correct?

(A) The surface density of bound charges on the inner and outer surfaces are $-k$ and $+k$, respectively. The volume density of bound charges inside the dielectric is zero.

(B) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $+k$.

(C) The surface density of bound charges on the inner and outer surfaces are $\frac{-k}{a^2}$ and $\frac{k}{b^2}$, respectively. The volume density of bound charges inside the dielectric is zero.

(D) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $\frac{3k}{4\pi(b^3 - a^3)}$.

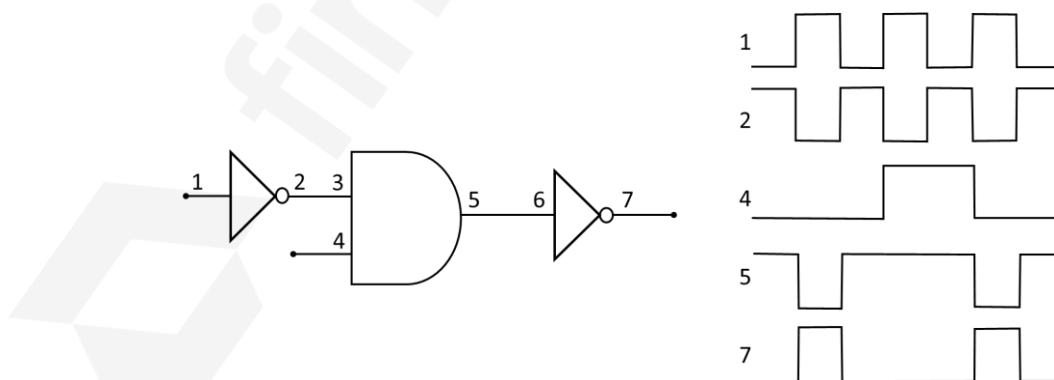
Q.38 One mole of an ideal gas having specific heat ratio (γ) of 1.6 is mixed with one mole of another ideal gas having specific heat ratio of 1.4. If C_V and C_P are the molar specific heat capacities of the gas mixture at constant volume and pressure, respectively, which of the following is/are correct? (R denotes the universal gas constant).

(A) $C_V = 2.08 R$ (B) $C_P = 2.9 R$ (C) $C_P = 1.48 C_V$ (D) $C_P = 1.52 C_V$

Q.39 Two relativistic particles with opposite velocities collide head-on and come to rest by sticking with each other. Which of the following quantities is/are conserved in the collision?

(A) Total momentum (B) Total energy
(C) Total kinetic energy (D) Total rest mass

Q.40 Figure shows a circuit diagram comprising Boolean logic gates and the corresponding timing diagrams show the digital signals at various points in the circuit. Which of the following is/are true?

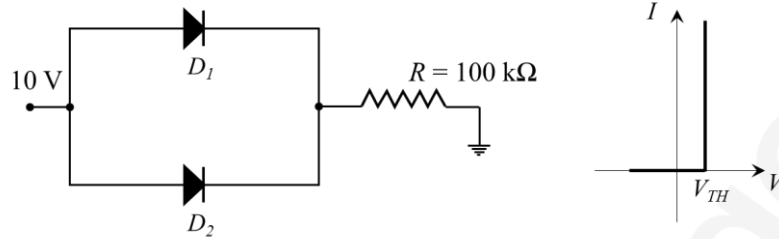


(A) Points 3 and 7 are shorted.
(B) The NOT gate on the right is faulty.
(C) The AND gate is faulty and acts like a NOR gate.
(D) The AND gate is faulty and acts like an OR gate.

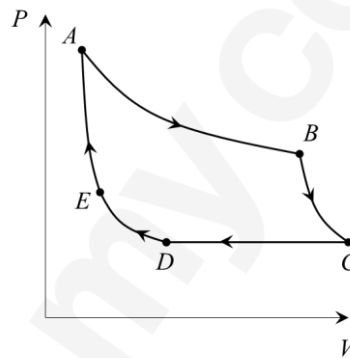
SECTION – C
NUMERICAL ANSWER TYPE (NAT)
Q. 41 – Q. 50 carry one mark each.

- Q.41 The line integral of the vector function $u(x, y) = 2y \hat{i} + x \hat{j}$ along the straight line from (0, 0) to (2, 4) is _____
- Q.42 Consider a thin bi-convex lens of relative refractive index $n = 1.5$. The radius of curvature of one surface of the lens is twice that of the other. The magnitude of larger radius of curvature in units of the focal length of the lens is _____ (Round off to 1 decimal place).
- Q.43 Water flows in a horizontal pipe in a streamlined manner at an absolute pressure of 4×10^5 Pa and speed of 6 m/s. If it exits the pipe at a pressure of 10^5 Pa, the speed of water at the exit point is _____ m/s (Round off to 1 decimal place).
(The density of water is 1000 kg/m^3)
- Q.44 Consider a retarder with refractive indices $n_e=1.551$ and $n_o=1.542$ along the extraordinary and ordinary axes, respectively. The thickness of this retarder for which a left circularly polarized light of wavelength 600 nm will be converted into a right circularly polarized light is _____ μm . (Round off to 2 decimal places)
- Q.45 Using a battery, a 10 pF capacitor is charged to 50 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V, its capacitance is _____ pF.
- Q.46 Consider two spherical perfect blackbodies with radii R_1 and R_2 at temperatures $T_1 = 1000$ K and $T_2 = 2000$ K, respectively. They both emit radiation of power 1 kW. The ratio of their radii, R_1/R_2 is given by _____.
- Q.47 In a Compton scattering experiment, the wavelength of incident X-rays is 0.500 \AA . If the Compton wavelength λ_C is 0.024 \AA , the value of the longest wavelength possible for the scattered X-ray is _____ \AA . (Specify up to 3 decimal places)
- Q.48 A solid with FCC crystal structure is probed using X-rays of wavelength 0.2 nm. For the crystallographic plane given by (2, 0, 0), a first order diffraction peak is observed for a Bragg angle of 21° . The unit cell size is _____ nm. (Round off to 2 decimal places)

- Q.49 The figure shows a circuit containing two diodes D_1 and D_2 with threshold voltages V_{TH} of 0.7 V and 0.3 V, respectively. Considering the simplified diode model, which assumes diode I - V characteristic as shown in the plot on the right, the current through the resistor R is _____ μA .



- Q.50 An ideal gas undergoes an isothermal expansion along a path AB, adiabatic expansion along BC, isobaric compression along CD, isothermal compression along DE, and adiabatic compression along EA, as shown in the figure. The work done by the gas along the process BC is 10 J. The change in the internal energy along process EA is 16 J. The absolute value of the change in the internal energy along the process CD is _____ J.



Q. 51 – Q. 60 carry two marks each.

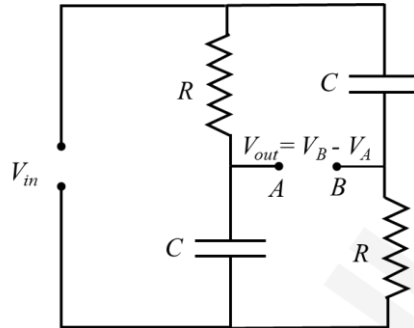
- Q.51 If a function $y(x)$ is described by the initial-value problem, $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$, with initial conditions $y(0) = 2$, and $\left(\frac{dy}{dx}\right)_{x=0} = 0$, then the value of y at $x = 1$ is _____ .
(Round off to 2 decimal places)

- Q.52 A vehicle of mass 600 kg with an engine operating at constant power P accelerates from rest on a straight horizontal road. The vehicle covers a distance of 600 m in 1 minute. Neglecting all losses, the magnitude of P is _____ kW. (Round off to 2 decimal places)

- Q.53 The angular momentum of a particle relative to origin varies with time (t) as $\vec{L} = (4\hat{x} + \alpha t^2\hat{y}) \text{ kg m}^2/\text{s}$, where $\alpha = 1 \text{ kg m}^2/\text{s}^3$. The angle between \vec{L} and the torque acting on the particle becomes 45° after a time of _____ s.

Q.54 Two transverse waves $y_1 = 5 \cos(kx - \omega t)$ cm, and $y_2 = 5 \cos(kx + \omega t)$ cm, travel on a string along x -axis. If the speed of a point at $x = 0$ is zero at $t = 0$ s, 0.25 s and 0.5 s, then the minimum frequency of the waves is _____ Hz.

Q.55 For the ac circuit shown in the figure, $R = 100 \text{ k}\Omega$ and $C = 10 \text{ pF}$, the phase difference between V_{in} and V_{out} is 90° at the input signal frequency of _____ kHz. (Round off to 2 decimal places)

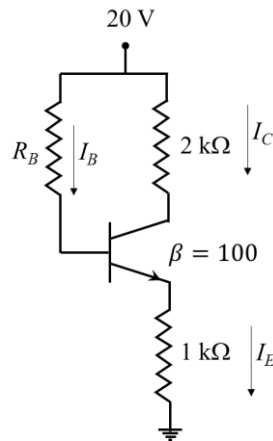


Q.56 The magnetic fields in tesla in the two regions separated by the $z = 0$ plane are given by $\vec{B}_1 = 3\hat{x} + 5\hat{z}$ and $\vec{B}_2 = \hat{x} + 3\hat{y} + 5\hat{z}$. The magnitude of the surface current density at the interface between the two regions is $\alpha \times 10^6 \text{ A/m}$. Given the permeability of the free space $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$, the value of α is _____. (Round off to 2 decimal places)

Q.57 A body at a temperature T is brought into contact with a reservoir at temperature $2T$. Thermal equilibrium is established at constant pressure. The heat capacity of the body at constant pressure is C_p . The total change in entropy of the body and the reservoir in units of C_p is _____. (Round off to 2 decimal places)

Q.58 One mole of an ideal monatomic gas at pressure P , volume V and temperature T is expanded isothermally to volume $4V$. Thereafter, the gas is heated isochorically (at constant volume) till its pressure becomes P . If R is the universal gas constant, the total heat transfer in the process, in units of RT is _____. (Round off to 2 decimal places)

- Q.59 In the transistor circuit given in the figure, the emitter-base junction has a voltage drop of 0.7 V. A collector-emitter voltage of 14 V reverse biases the collector. Assuming the collector current to be the same as the emitter current, the value of R_B is _____ $k\Omega$.



- Q.60 The radioactive nuclei ^{40}K decay to ^{40}Ar with a half-life of 1.25×10^9 years. The $^{40}\text{K}/^{40}\text{Ar}$ isotopic ratio for a particular rock is found to be 50. The age of the rock is $m \times 10^7$ years. The value of m is _____. (Round off to 2 decimal place)

END OF THE QUESTION PAPER