NG 17 (GROUP A)

PART AA — ENGINEERING MATHEMATICS

(Common to all candidates)

(Answer ALL questions)

1. The system of linear equations $4x+3y=7$, $2x+y=6$ has
   1. a unique solution
   2. no solution
   3. an infinite number of solutions
   4. exactly two distinct solutions

2. Let $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$. The eigenvalues of $2A^{-1}$ are
   1. $-\frac{1}{3}$ and -2
   2. $\frac{1}{2}$ and $\frac{1}{3}$
   3. -1 and -6
   4. 3 and $\frac{1}{2}$

3. The quadratic form $Q(x,y)=3x^2+2xy+4y^2$ is
   1. positive semidefinite
   2. negative semidefinite
   3. negative definite
   4. positive definite

4. Let $u(x,y)=\log\left(\frac{x^2}{y}\right)$. The value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to
   1. $2u$
   2. 1
   3. 0
   4. u

5. The particular integral of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=e^x\cos x$ is
   1. $\frac{x^2e^x\sin x}{2}$
   2. $\frac{xe^x\sin x}{3}$
   3. $\frac{xe^x\sin x}{2}$
   4. $\frac{x^2e^x\sin x}{3}$

6. By eliminating the constants 'a' and 'b' from $x^2+y^2+(z-a)^2=b^2$, the partial differential equation is
   1. $x^2\frac{\partial z}{\partial y}-y^2\frac{\partial z}{\partial x}=0$
   2. $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=0$
   3. $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$
   4. $x\frac{\partial z}{\partial y}-y\frac{\partial z}{\partial x}=0$

7. If ϕ and ψ are scalar functions, then the value of $\nabla\cdot(\nabla\phi\times\nabla\psi)$ is

1. 1
2. 0
3. –1
4. 2

8. Let $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$ and S be the surface of a unit sphere. By the Gauss divergent theorem, the value of $\iint_S \vec{F}\cdot\hat{n}\,dS$, where $\hat{n}$ is a unit outward normal to S, is

1. 2π
2. $\frac{4\pi}{3}$
3. 4π
4. $\frac{5\pi}{3}$

9. If $f(z)=u(x,y)+iv(x,y)$ is analytic in a domain D, then its component functions $u(x,y)$ and $v(x,y)$ are

1. harmonic in D
2. not harmonic in D
3. not satisfying the C-R equations in D
4. not differentiably partially in D

10. The residue of $f(z)=\frac{ze^z}{(z-1)^3}$ is

1. 1
2. $\frac{3e}{2}$
3. $\frac{2e}{3}$
4. $\frac{e}{2}$

11. The Laurent expansion of $f(z)=\frac{1}{z(z-1)}$ valid for $|z|>1$ is

1. $\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right)$
2. $\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right)-\frac{1}{z}$
3. $z\left(1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right)-\frac{1}{z}$
4. $z\left(1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right)$

12. Let $F(s)=\frac{1}{s(s^2+1)}$ be the Laplace transform of $f(t)$. By inverse Laplace transform, $f(t)$ is

1. $1-\sin t$
2. $1-\cos t$
3. $1+\cos t$
4. $1+\sin t$

13. The Fourier cosine transform of $f(x)=e^{-x}$, $x>0$ is

1. $\sqrt{\frac{2}{\pi}}\left(\frac{1}{1+s^2}\right)$
2. $\sqrt{\frac{\pi}{2}}\left(\frac{1}{1+s^2}\right)$
3. $\sqrt{\frac{2}{\pi}}\left(\frac{s}{1+s^2}\right)$
4. $\sqrt{\frac{\pi}{2}}\left(\frac{s}{1+s^2}\right)$